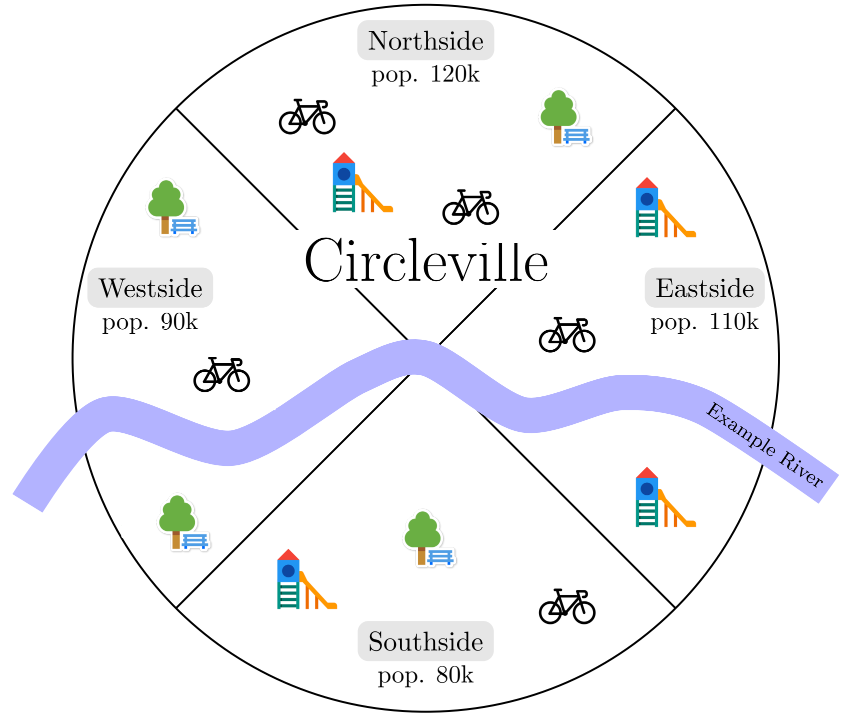
Computational Social Choice

Participatory Budgeting

Piotr Skowron

University of Warsaw



The model

Elements of the model:

1. A set of *candidates* or *projects* $C = \{c_1, c_2, \ldots, c_m\}$.
   Each candidate c comes with a cost, $\mathrm{cost}(c)$.

2. There is a budget constraint b:
   We have to select a subset of projects W s.t. $\sum_{c \in W} \mathrm{cost}(c) \leq b$.

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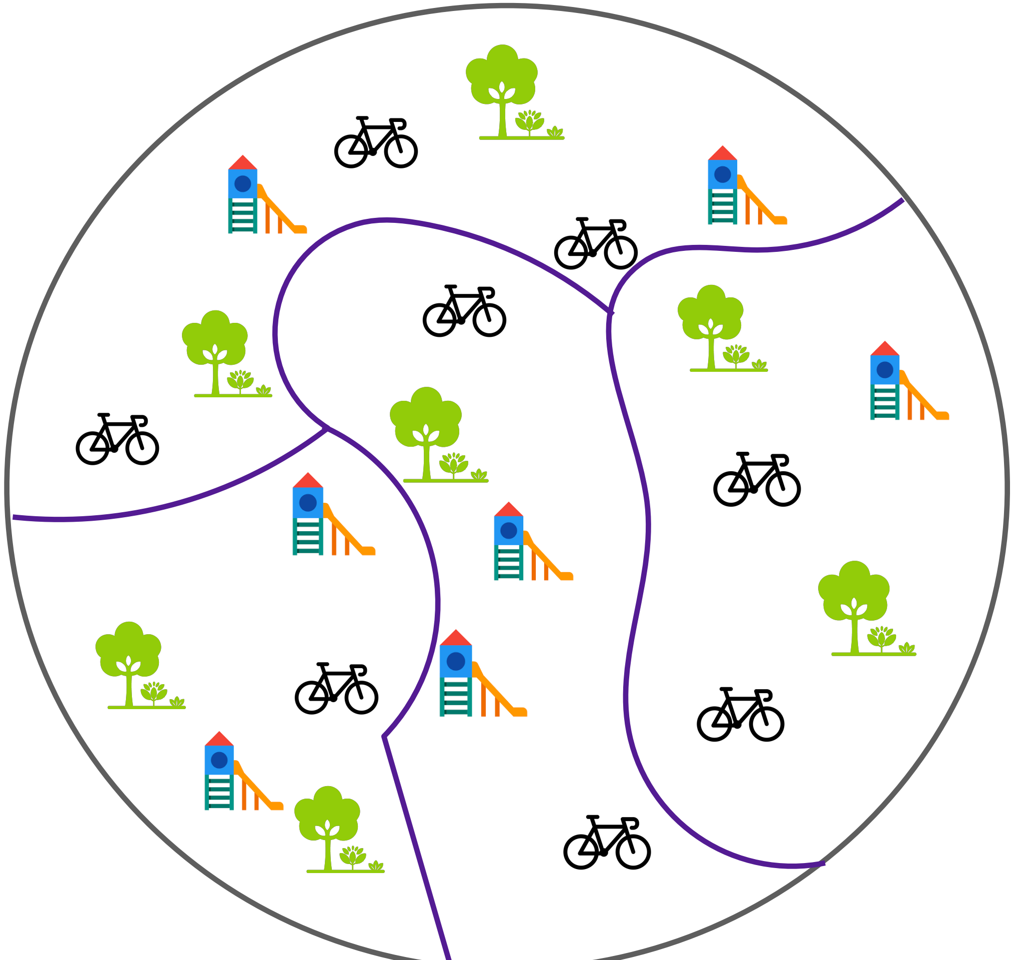
3. A set of voters $N = \{1, 2, \ldots, n\}$.
   Each voter has preferences over the projects.

How this is currently done



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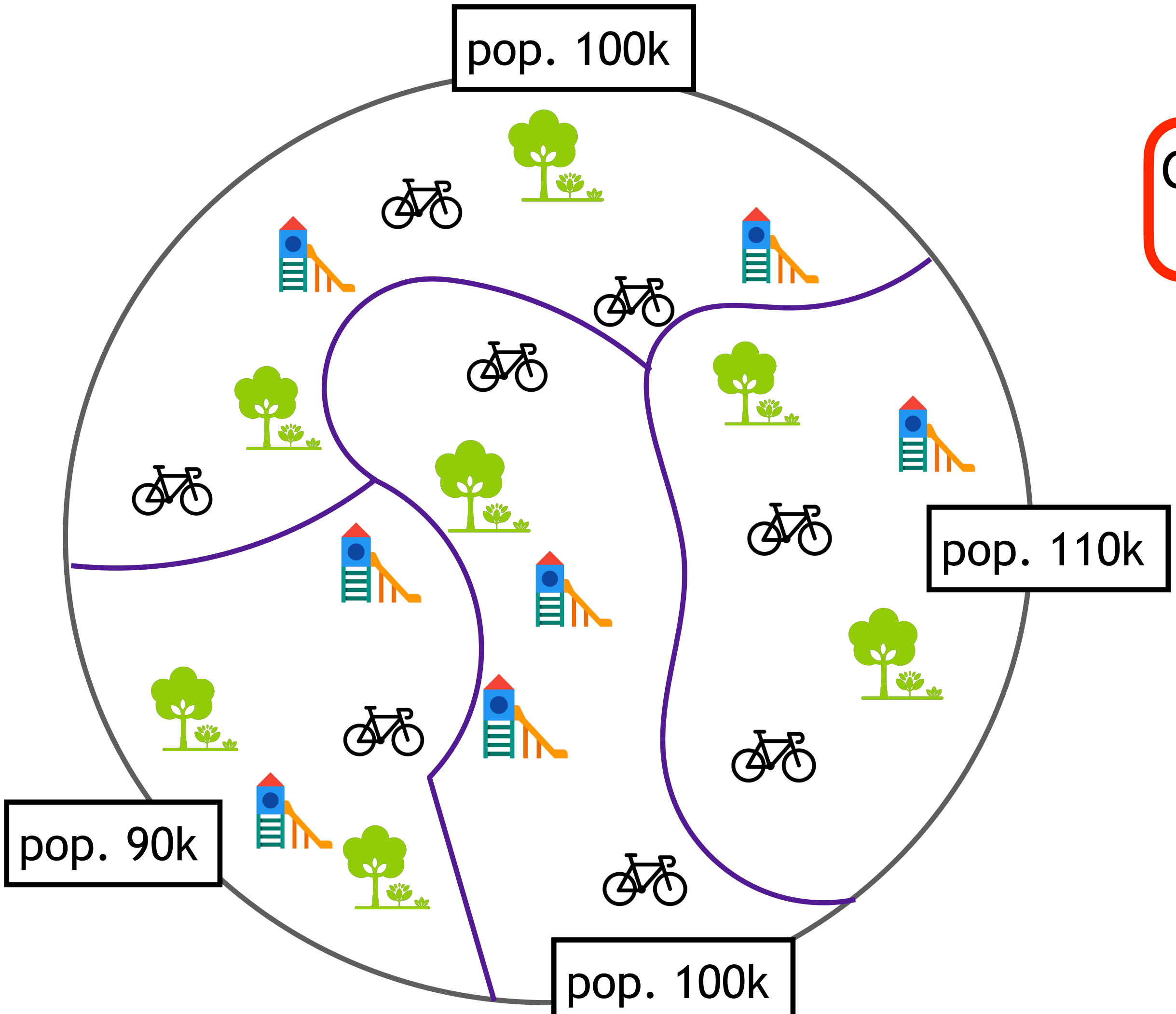
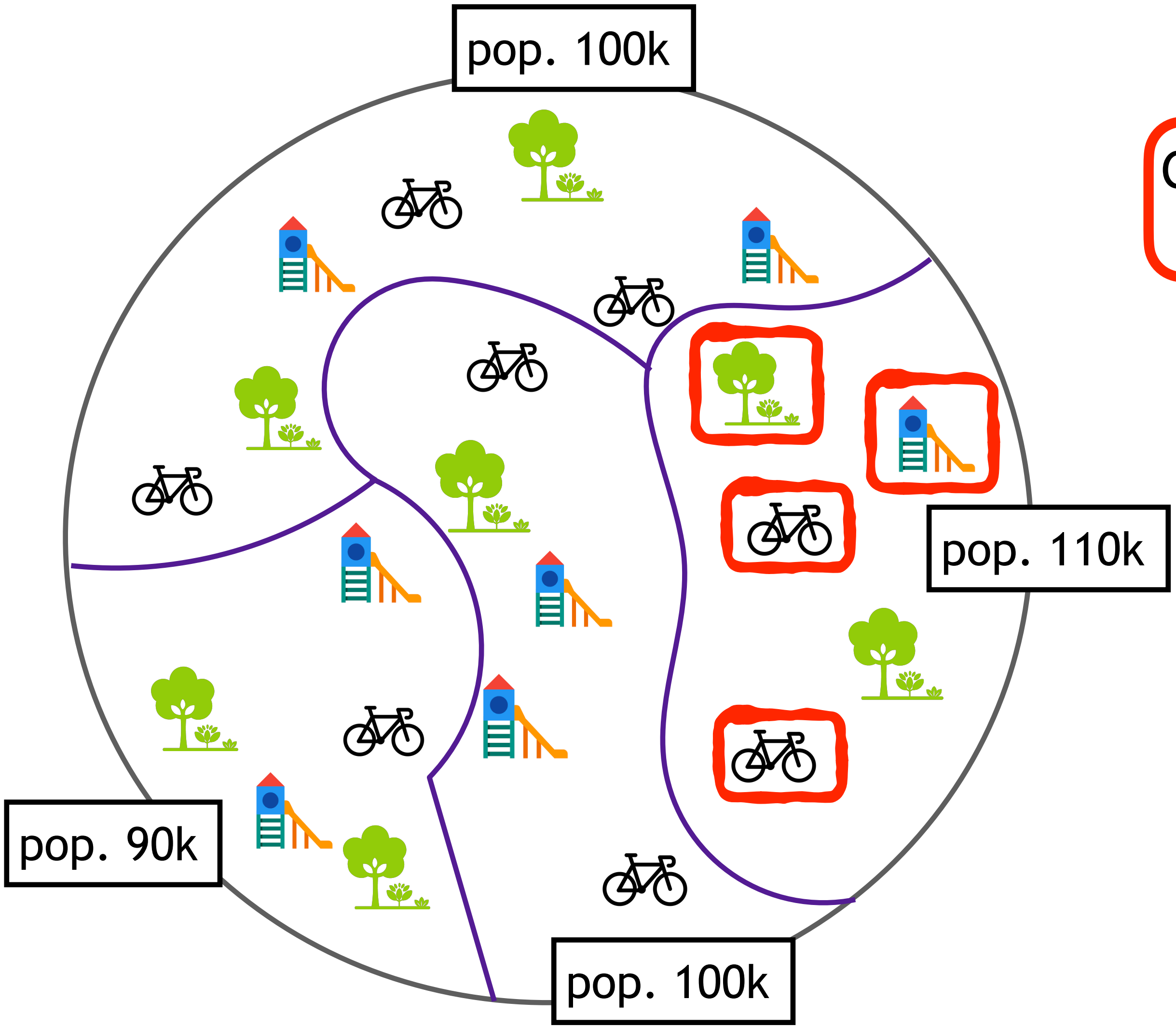


Choosing by the number of votes

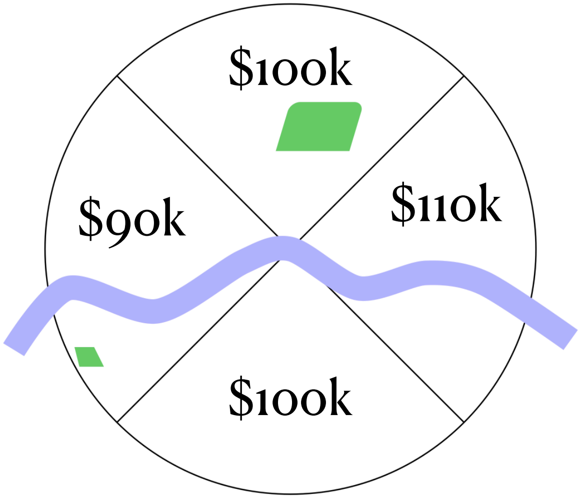
How this is currently done



Choosing by the number of votes

How this is currently done

Solution: Divide the budget upfront between the districts!



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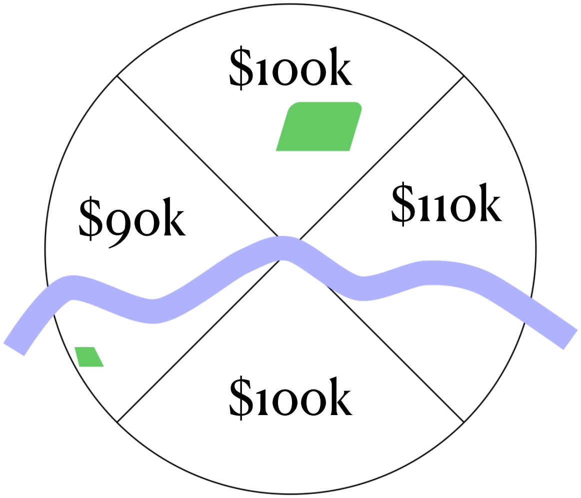

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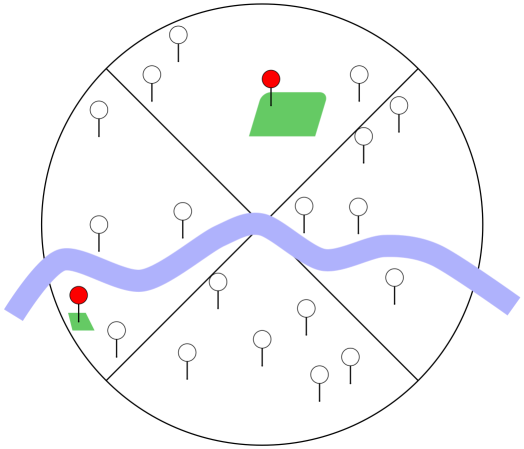

parents who want
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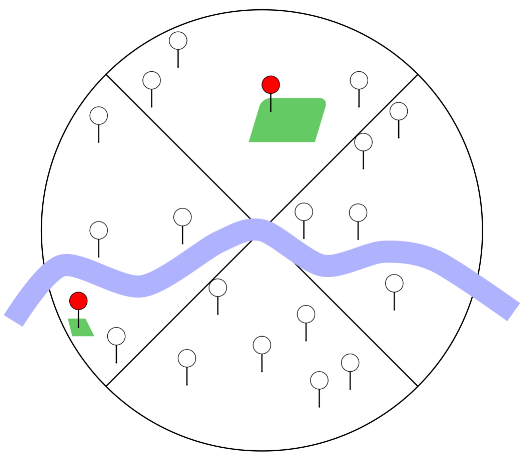

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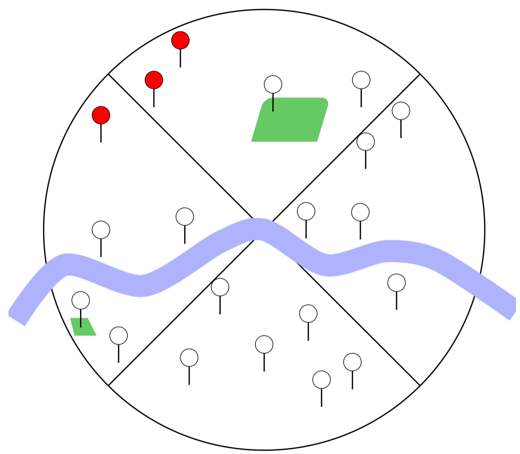

voters close to
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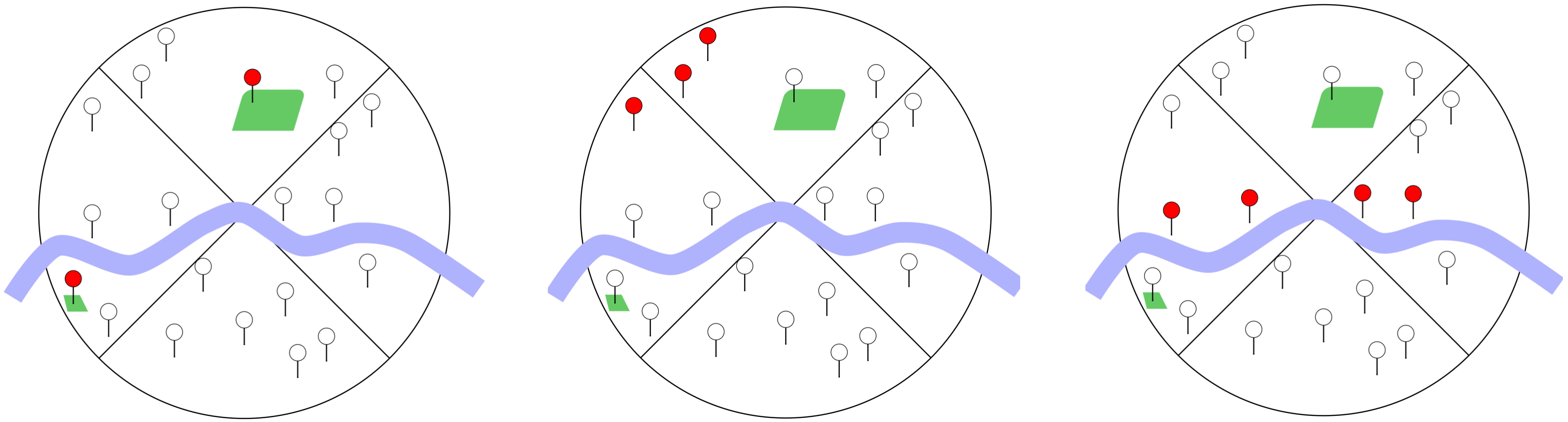

parents who want a playground

voters close to the border

cyclists who want a bike trail

How this is currently done?

A district where all the submitted projects have low support still needs to fund such unpopular projects.

	Project	Votes	Cost	Selected
Krakow 2021	Green areas in Prądnik **(Citywide)**	**3 177** (2101 from Prądnik)	300k	**NO**
Krakow 2021	Park in Olszy **(Prądnik)**	**1347**	550k	**YES**

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Warsaw 2021	Plants along Modlińska street **(Citywide)**	**12 463** (4 365 from Białołęka)	435k	NO
Warsaw 2021	Pavement along Modlińska str. **(Białołęka)**	**1 932**	630k	YES

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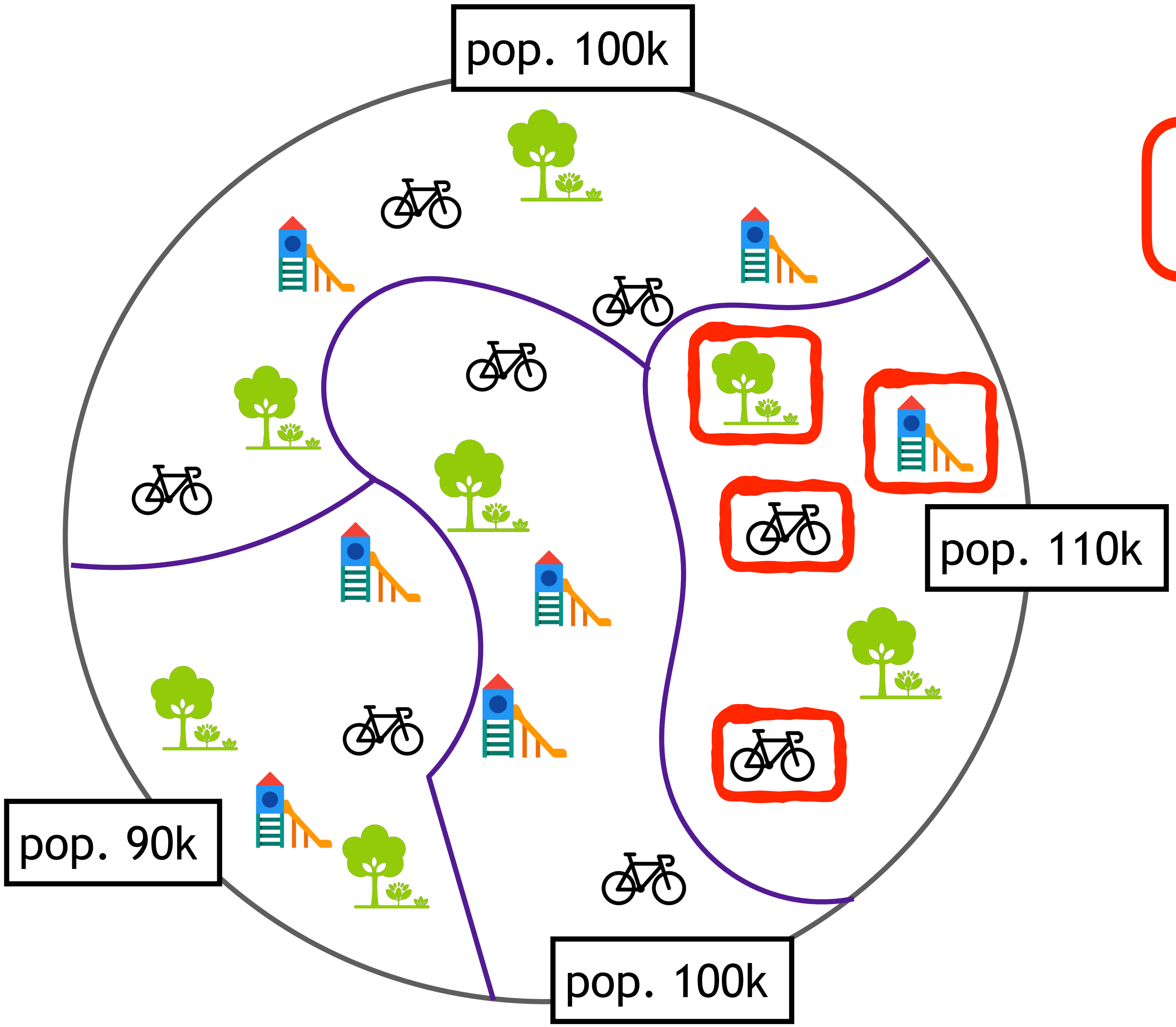
Warsaw 2020	New plants at Muranów **(Citywide)**	**5 623** (1 228 from Wola)	293k	**NO**
Warsaw 2020	Lamps and plants at Pustola str. **(Wola)**	**785**	310k	**YES**

How this is currently done



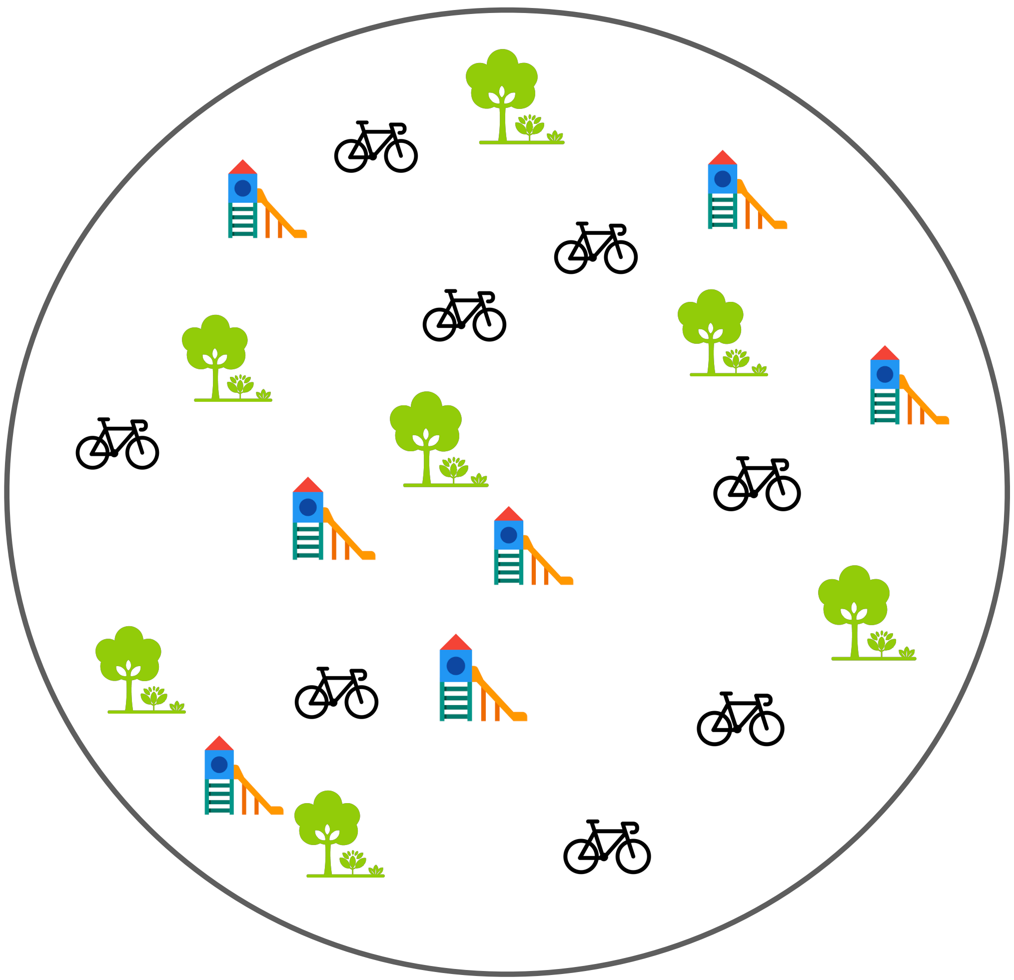
Districts are not the only division of voters

How this is currently done

30% voters (green areas)

30% voters (playgrounds)

40% voters (bike infrastructure)



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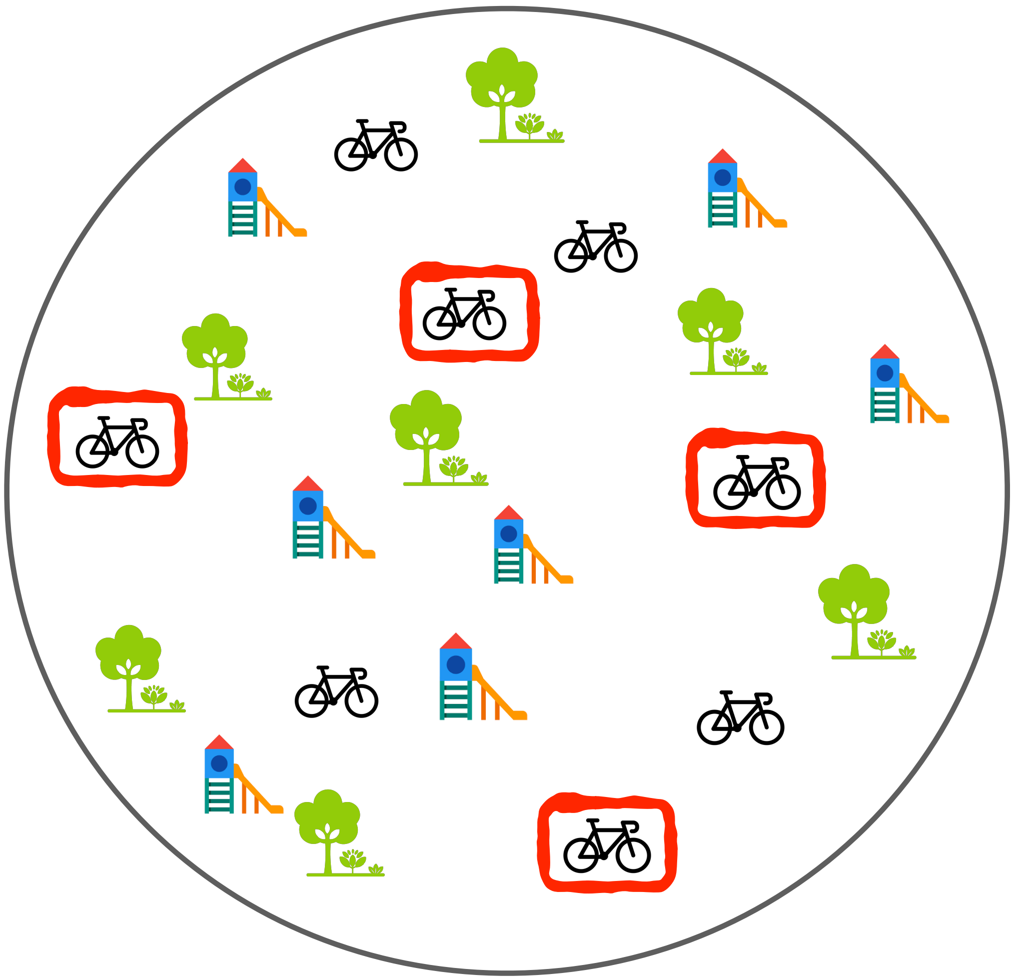

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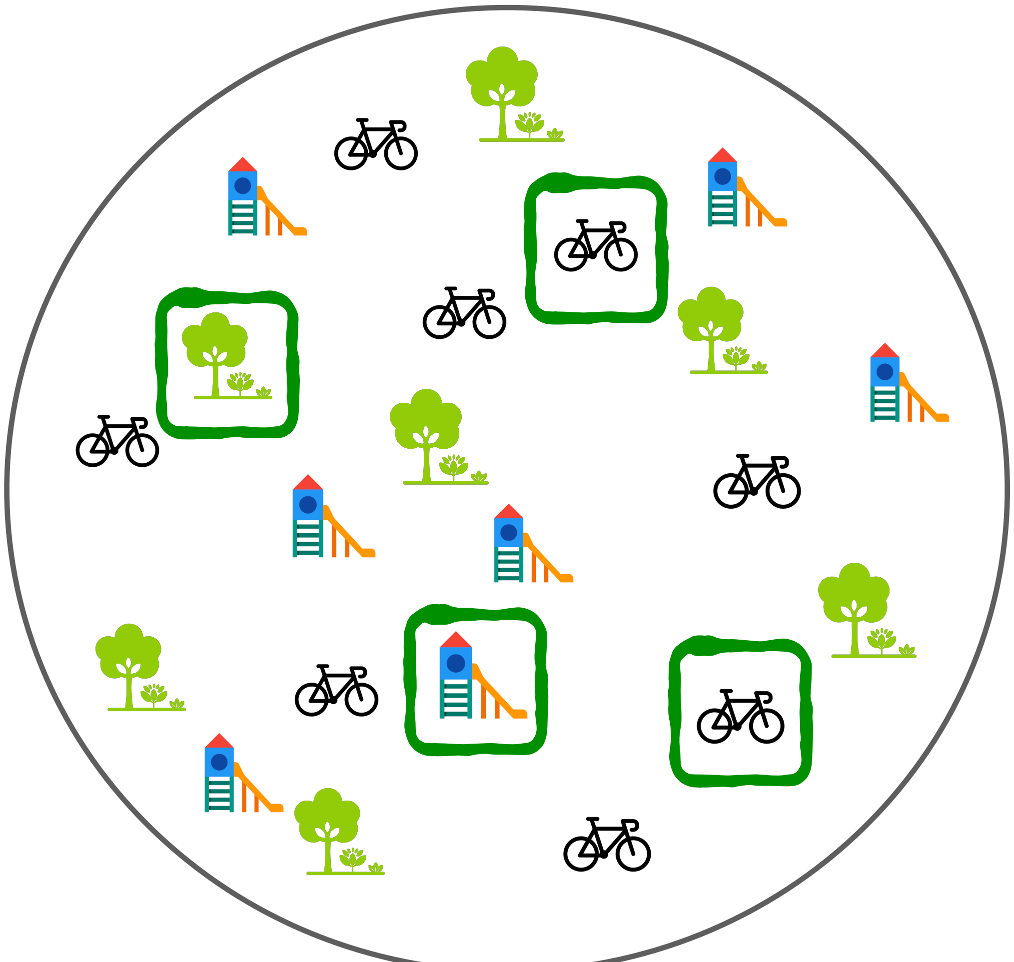
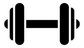


Choosing by the number of votes

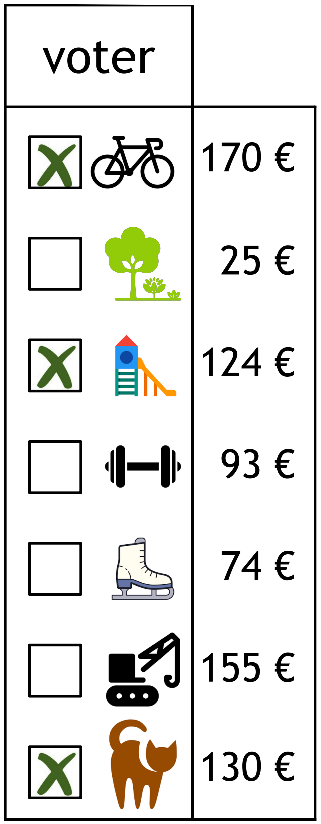
The rule should be fair to all groups of voters

Criterion of fairness.

voter		
X		170 €
		25 €
X		124 €
		93 €
		74 €
		155 €
X		130 €

$A(i)$: a subset of projects that voter i approves.

Criterion of fairness.



Extended justified representation (EJR):
We say that a group of voters S is T-cohesive for $T \subseteq C$ if

$$\frac{\mathrm{cost}(T)}{|S|} \leq \frac{b}{n} \text{ and } T \subseteq \bigcap_{i \in S} A(i).$$

A rule $\mathscr{R}$ satisfies extended justified representation if for each election instance E and each T-cohesive group S of voters there exists a voter $i \in S$ such that

$$|A(i) \cap \mathscr{R}(E)| \geq |T|.$$

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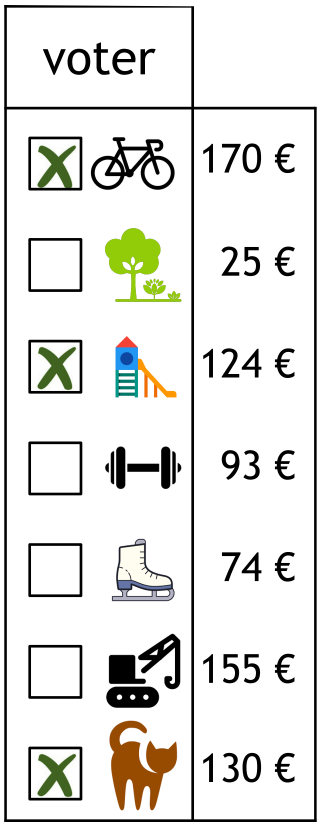

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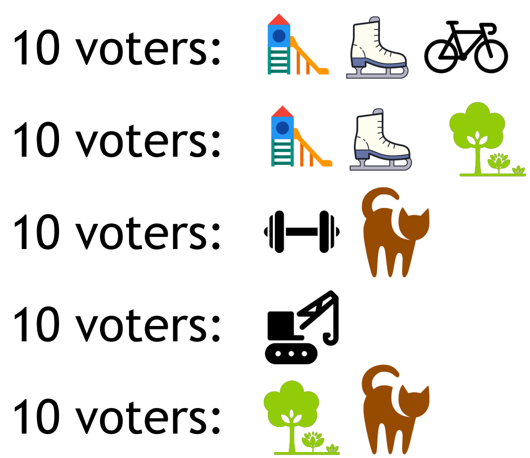

$b = 500$

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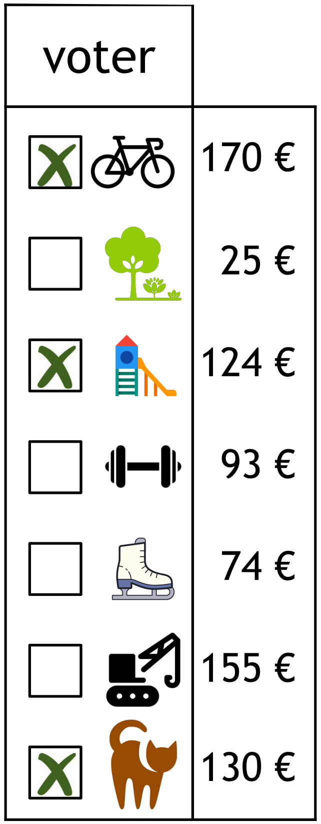

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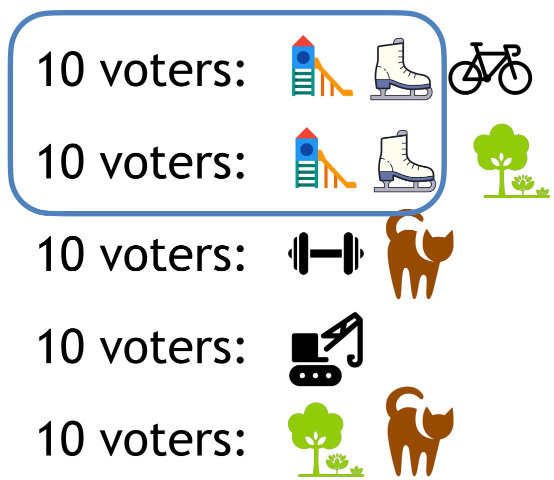

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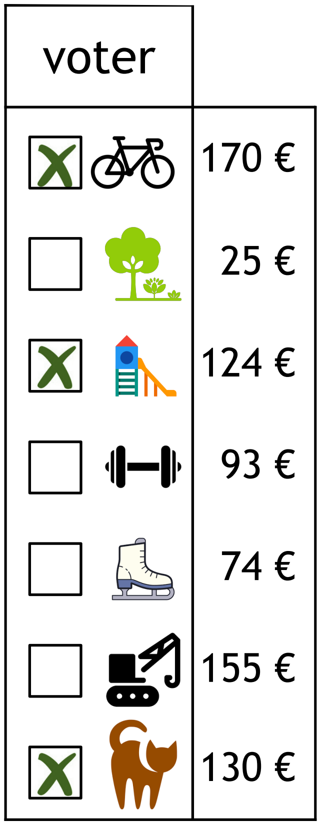

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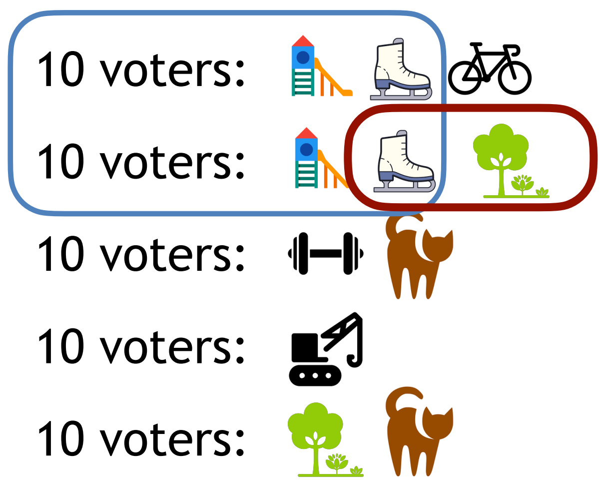

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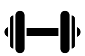
Ideally it should work for cardinal utilities

voter	
4 	170 €
2 	25 €
9 	124 €
7 	93 €
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We say that a group of voters S is (α, T)-cohesive for $\alpha : C \to \mathbb{R}$ and $T \subseteq C$ if:

$$\frac{\mathrm{cost}(T)}{|S|} \leq \frac{b}{n} \quad \text{and} \quad u_i(c) \geq \alpha(c) \text{ for all } i \in S, c \in T.$$

A rule $\mathscr{R}$ satisfies extended justified representation if for each election instance E and each (α, T)-cohesive group S of voters there exists a voter $i \in S$ such that

$$\sum_{c \in \mathscr{R}(E)} u_i(c) \geq \sum_{c \in T} \alpha(c).$$

$u_i(c)$: a utility that voter i assigns to c.

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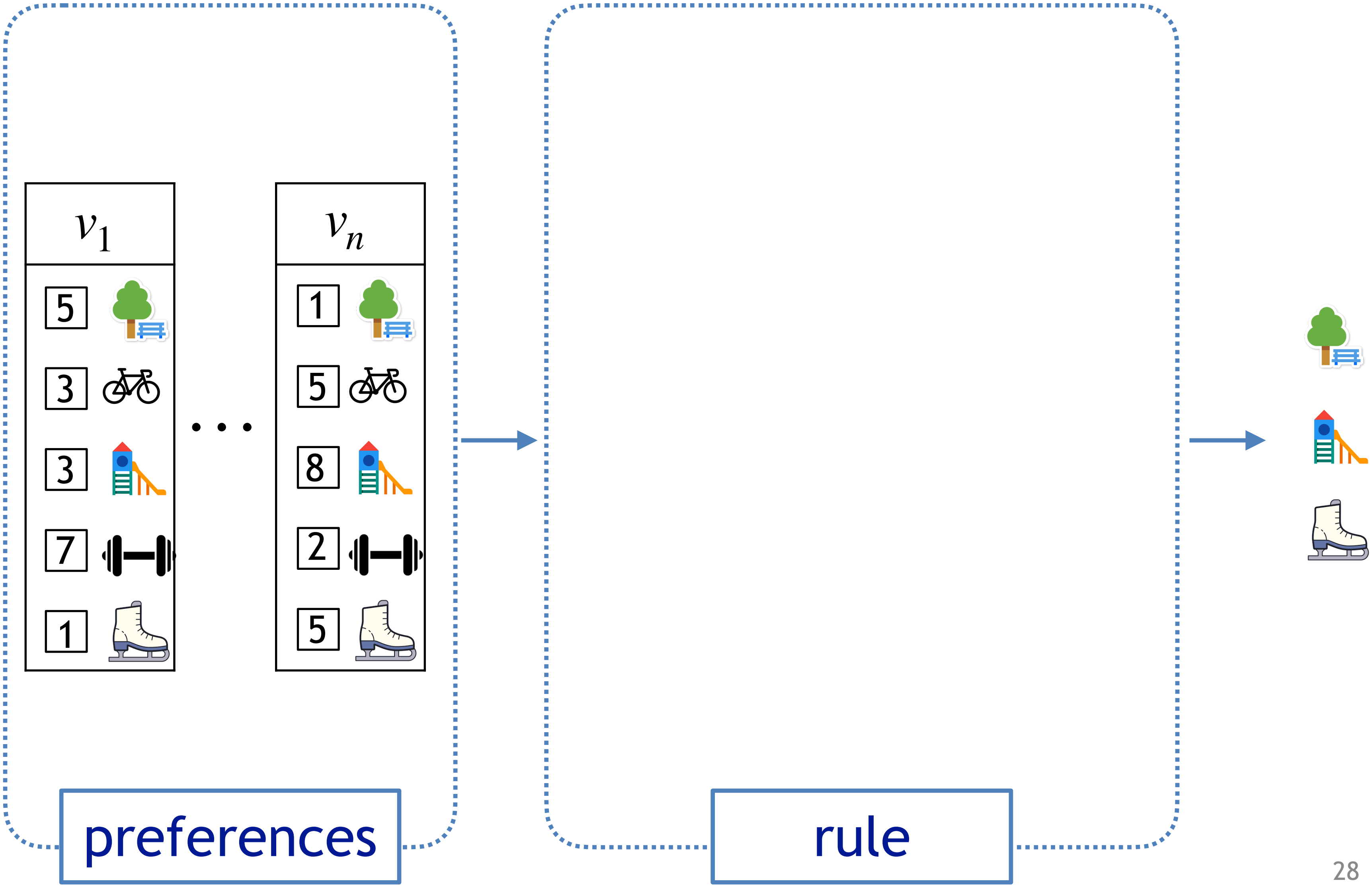
A rule $\mathcal{R}$ satisfies extended justified representation **up-to-one** if for each election instance E and each (α, T)-cohesive group S of voters there exists a voter $i \in S$ and a candidate $d \in C$ such that

$$u_i(d) + \sum_{c \in \mathcal{R}(E)} u_i(c) \geq \sum_{c \in T} \alpha(c).$$

Method of Equal Shares: Idea



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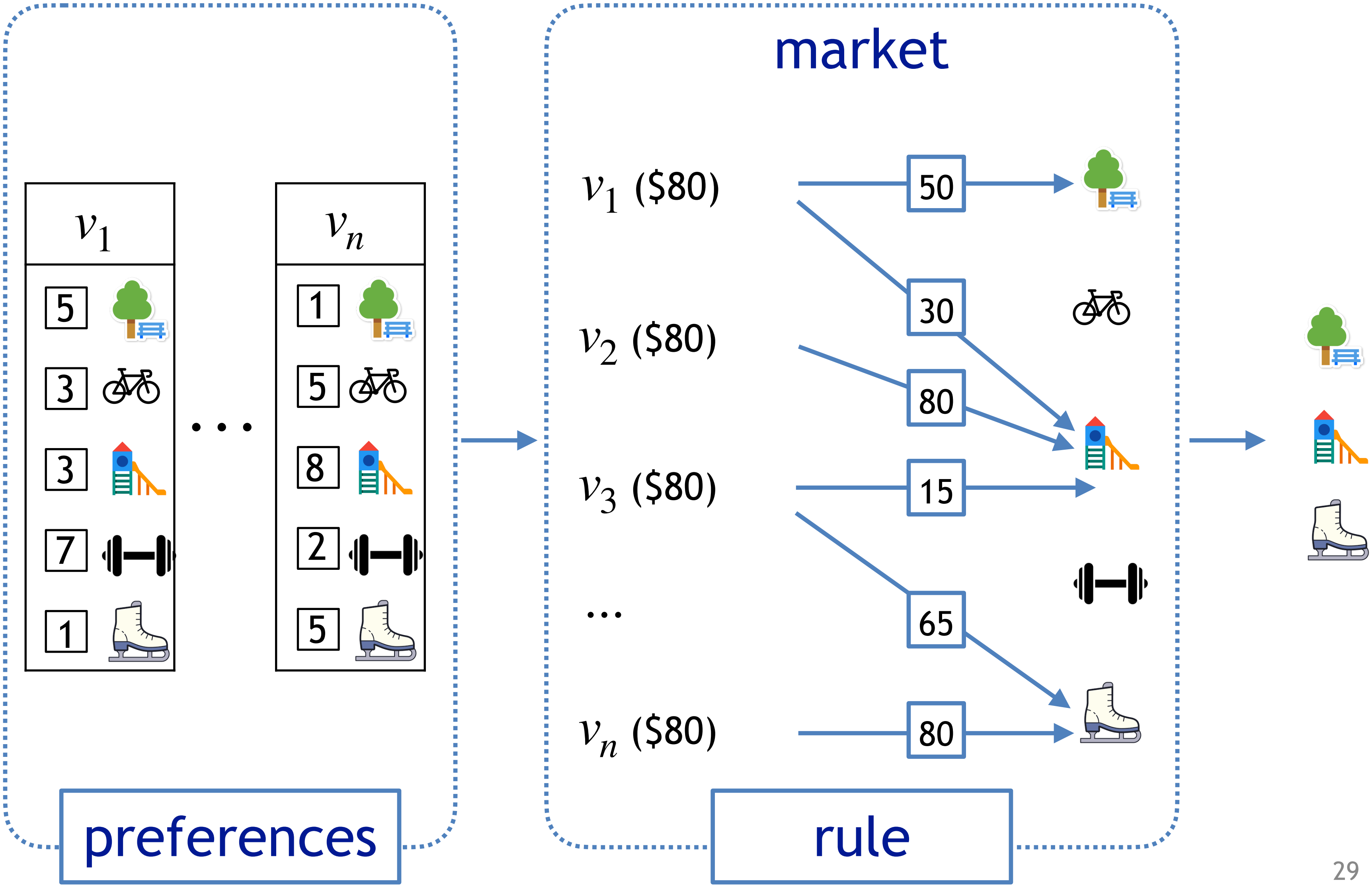

Method of Equal Shares for Approvals

1. The budget is evenly divided among the voters.
2. If a candidate $c \in C$ is selected its cost is divided among the voters who voted for c.
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$b = \$1000$

	(20 votes)	(26 votes)	(11 votes)	(9 votes)	(20 votes)	(14 votes)
A (costs $200)	✓	✓				✓
B (costs $200)	✓	✓				
C (costs $200)	✓	✓		✓		
D (costs $200)	✓	✓	✓	✓		
E (costs $200)	✓	✓				
F (costs $200)	✓			✓		✓
G (costs $200)			✓	✓		✓
H (costs $200)					✓	

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G (costs \$200)			✓	✓		✓
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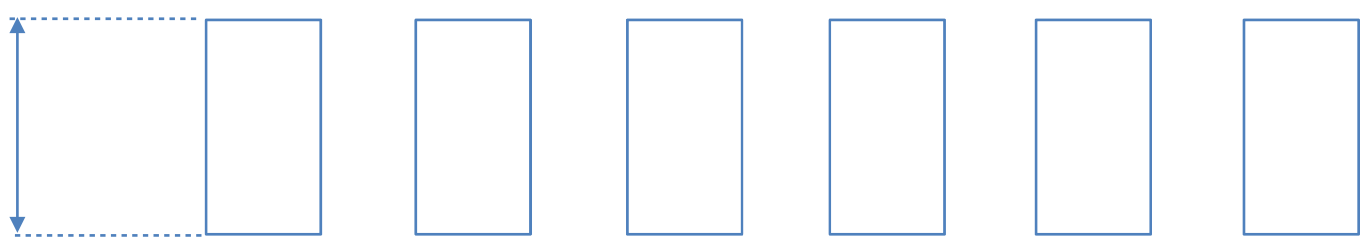
20 voters: 26 voters: 11 voters: 9 voters: 20 voters: 14 voters:

$10



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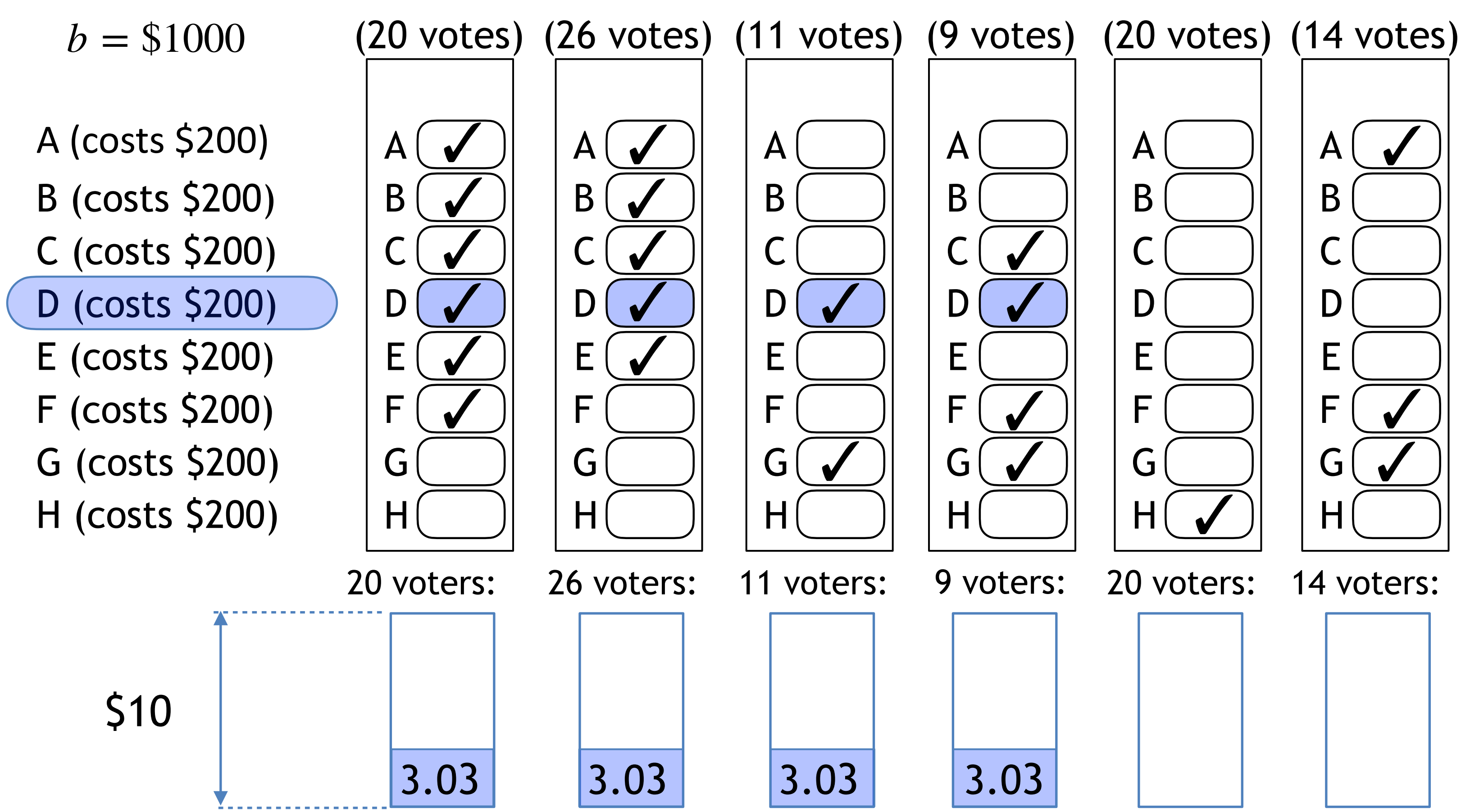

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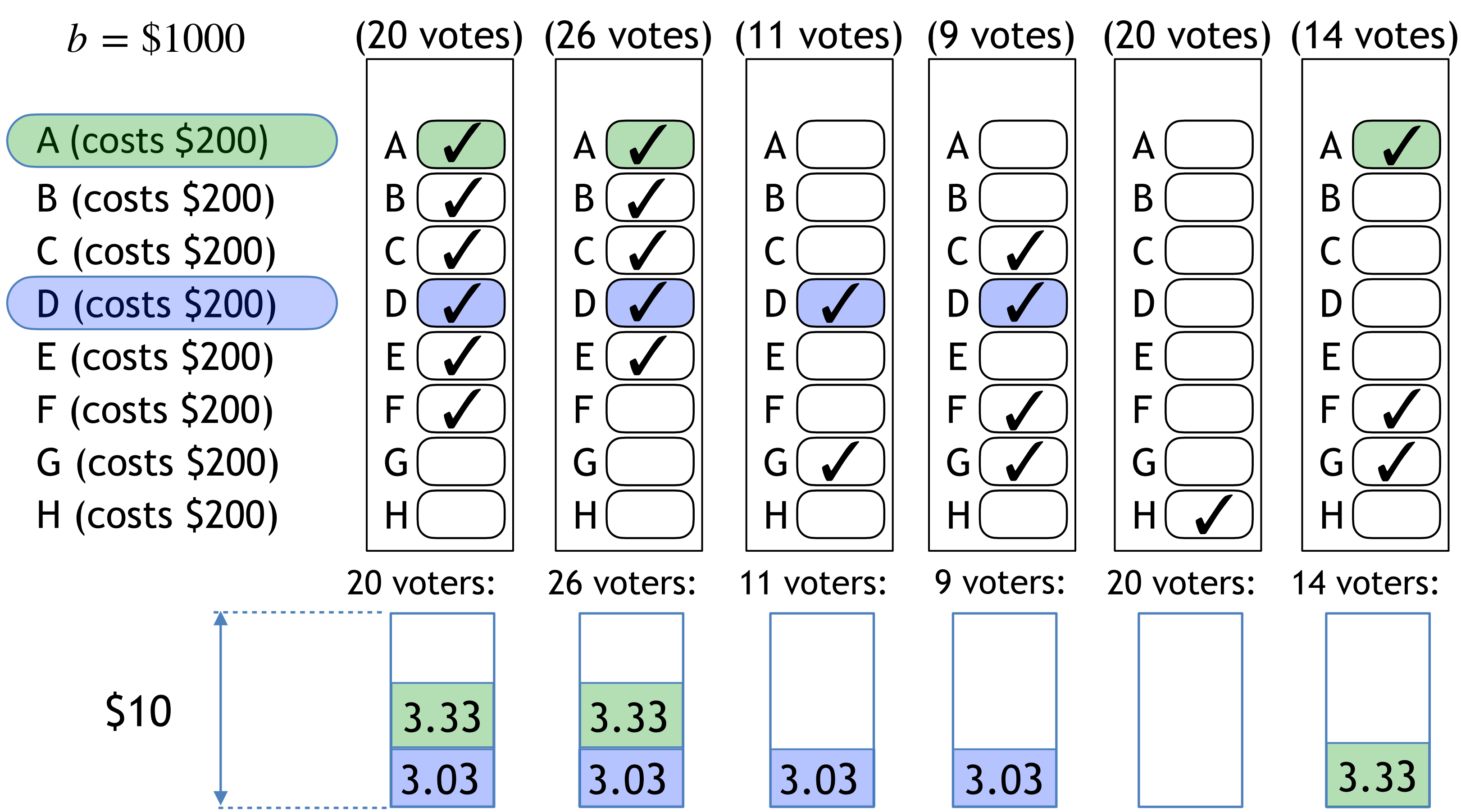

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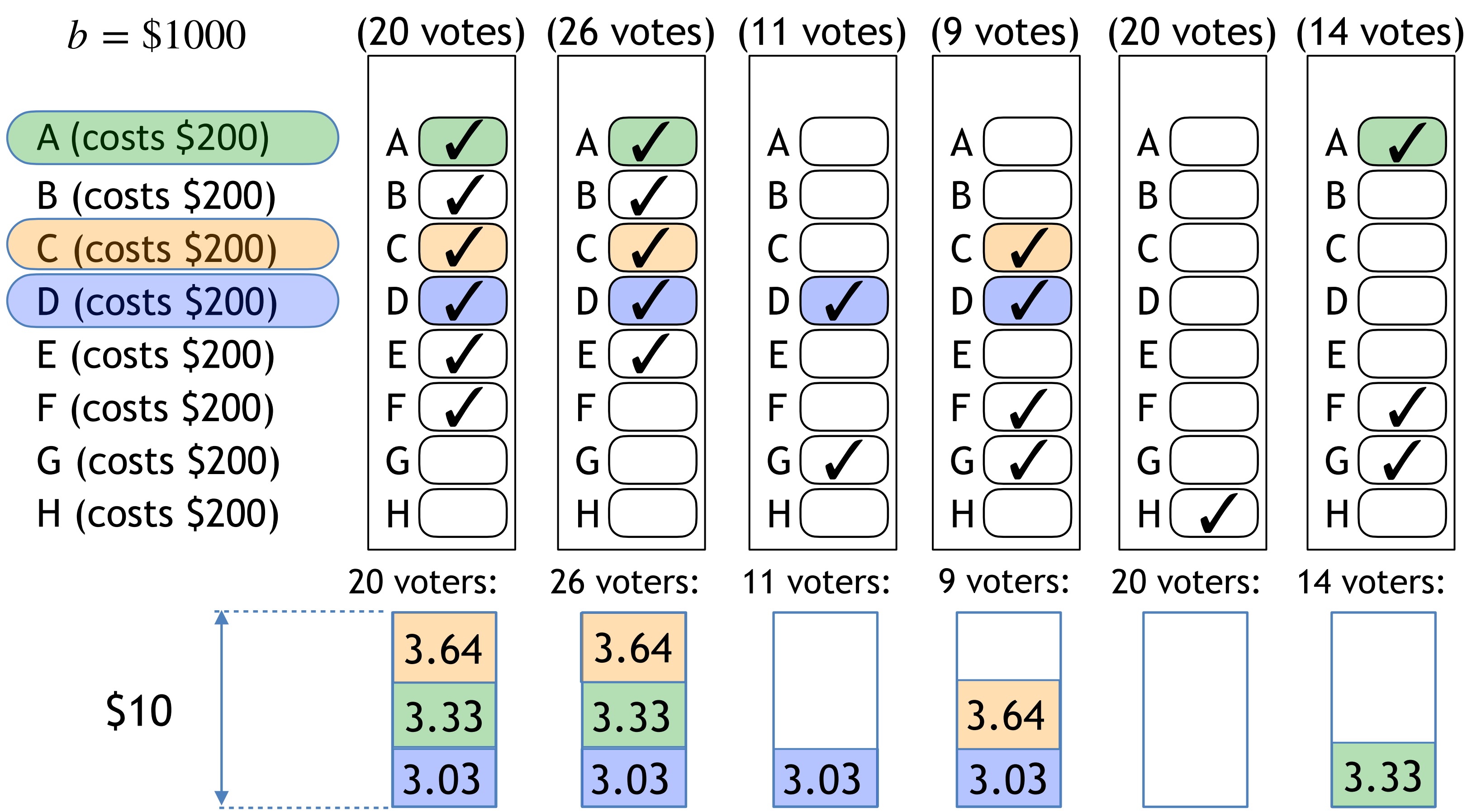

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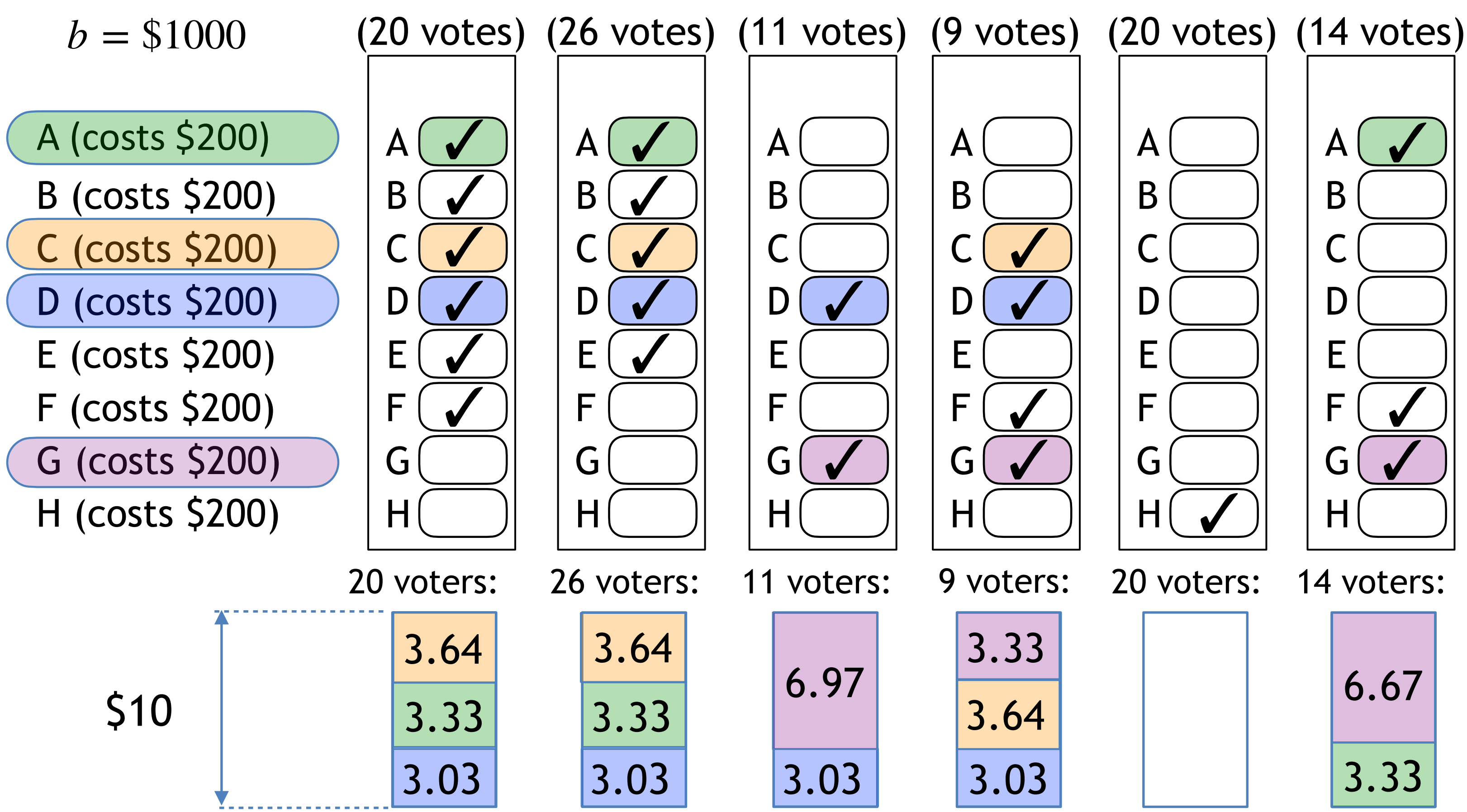

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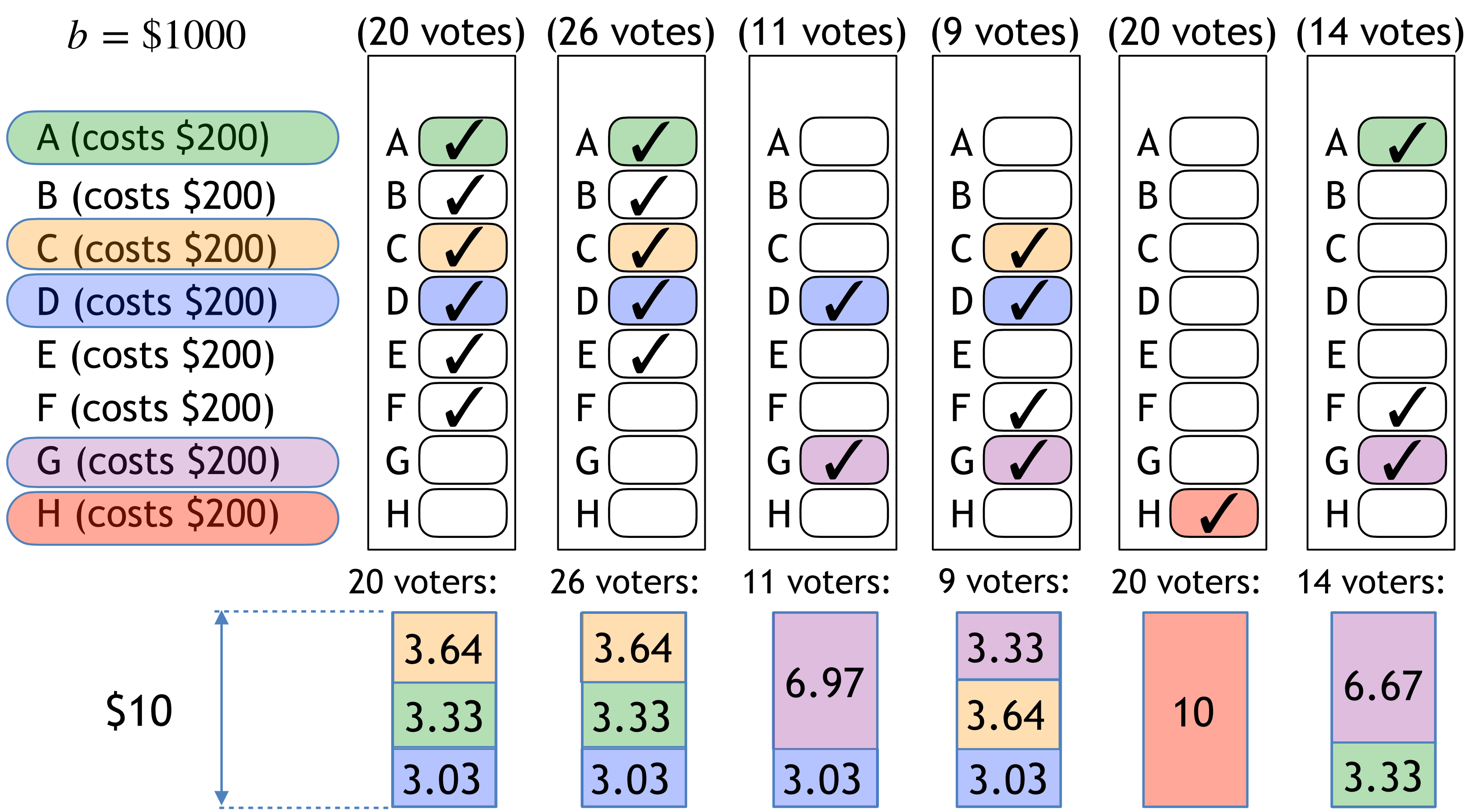

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Proof: W.l.o.g, assume that costs of all candidates equal to one. Consider a group of voters S and a group of candidates $T \subseteq C$ such that $|T|/|S| \leq b/n$ and $T \subseteq \bigcap_{i \in S} A(i)$.

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Proof: W.l.o.g, assume that costs of all candidates equal to one. Consider a group of voters S and a group of candidates $T \subseteq C$ such that $|T|/|S| \leq b/n$ and $T \subseteq \bigcap_{i \in S} A(i)$. Let W be the set returned by MES, and towards a contradiction, assume $|A(i) \cap W| < |T|$. First, we show that no voter in S paid more than $\frac{b}{n|T|}$ for any candidate. Towards a contradiction, consider first such a purchase. Before it, each voter in S paid at most: $(|T| - 1)\frac{b}{n|T|}$, and thus was left with at least $\frac{b}{n|T|}$ dollars.

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      To add a candidate c to W, we will need that $\sum_{i \in N} p_i(c) = \mathrm{cost}(c)$.
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      $$\sum_{i \in N} \min\left(\frac{b}{n} - \sum_{c' \in W} p_i(c'), u_i(c) \cdot \rho\right) = \mathrm{cost}(c)$$

   3. If no candidate is ρ-affordable for any ρ, the rule returns W.
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$b = \$2500$

Project	(65 votes)	(35 votes)	(35 votes)	(50 votes)	(10 votes)	(55 votes)
A (costs $120)				1	2	
B (costs $200)	30	30				
C (costs $500)		30			5	10
D (costs $600)				100		
E (costs $500)	10		30			
F (costs $180)			10	10		10
G (costs $1000)	10	10	40	100	40	40
H (costs $110)				2	1	1

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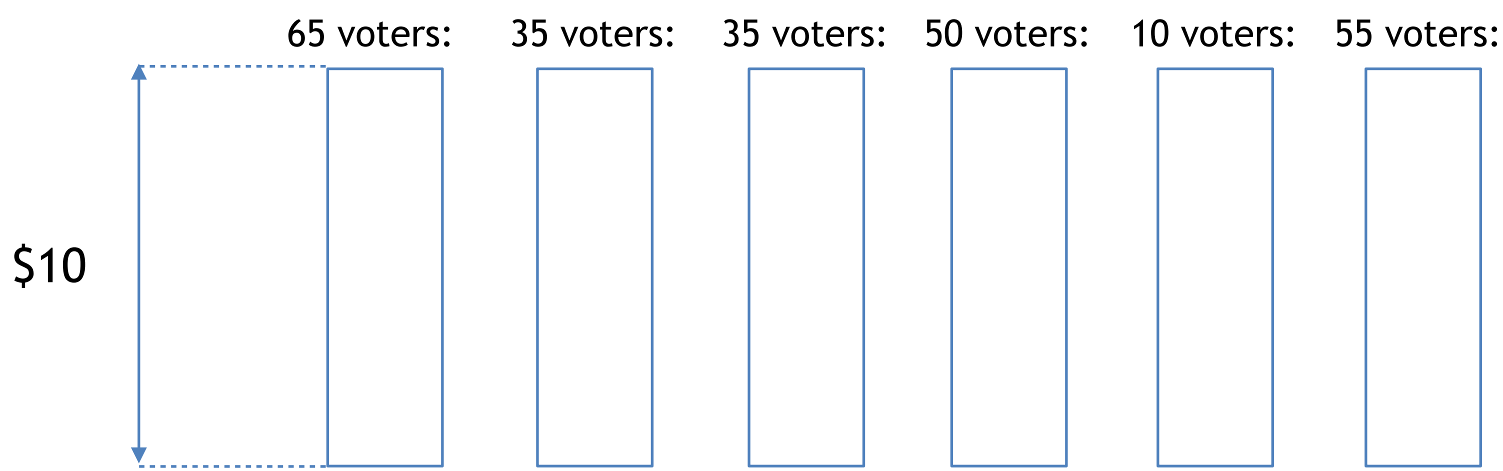

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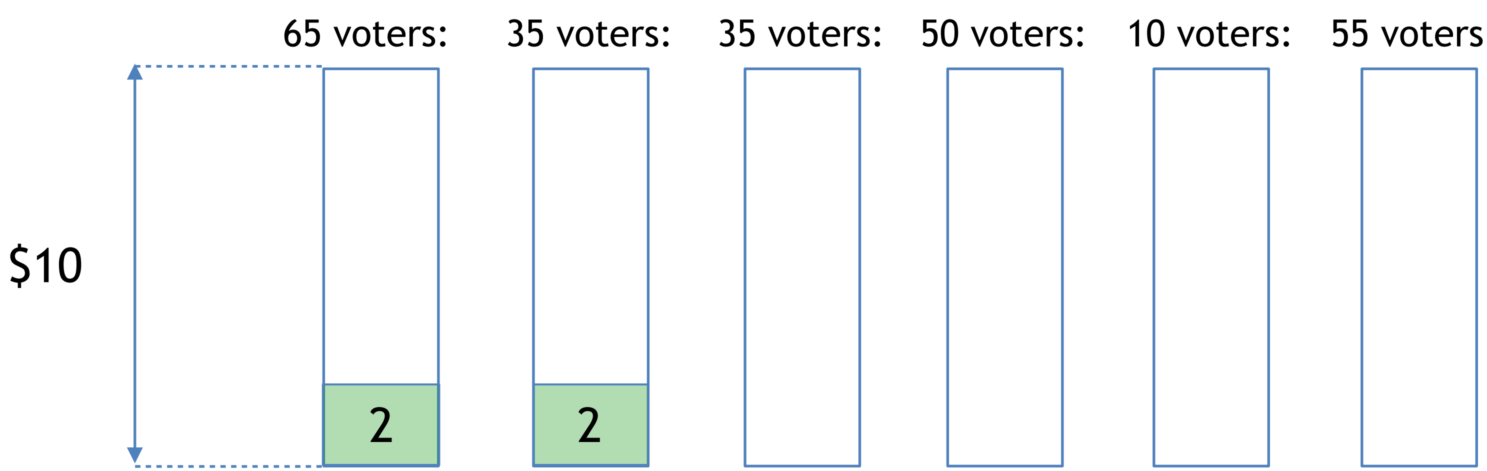

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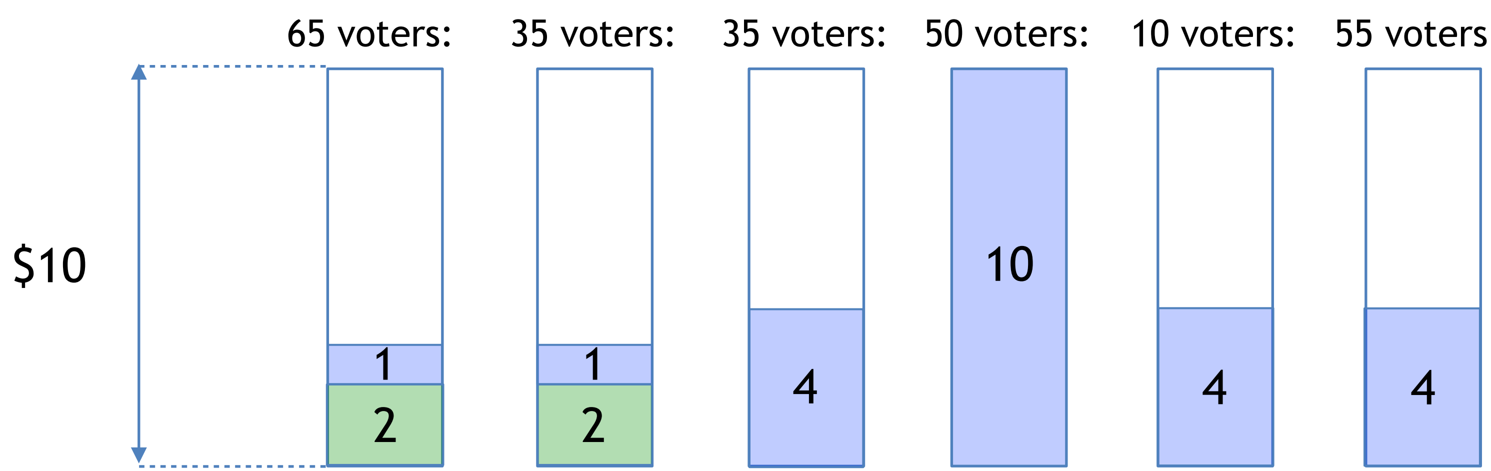

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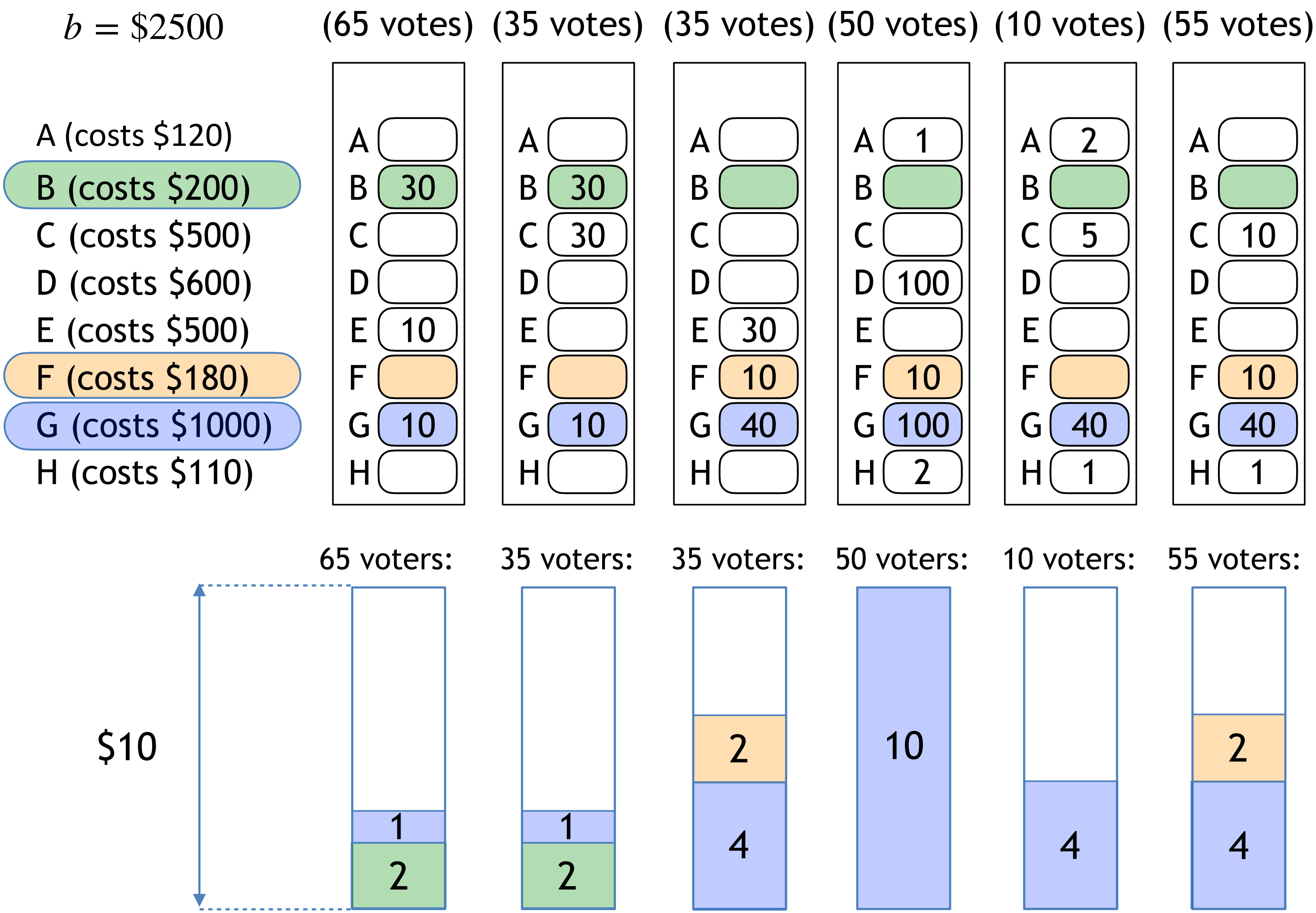

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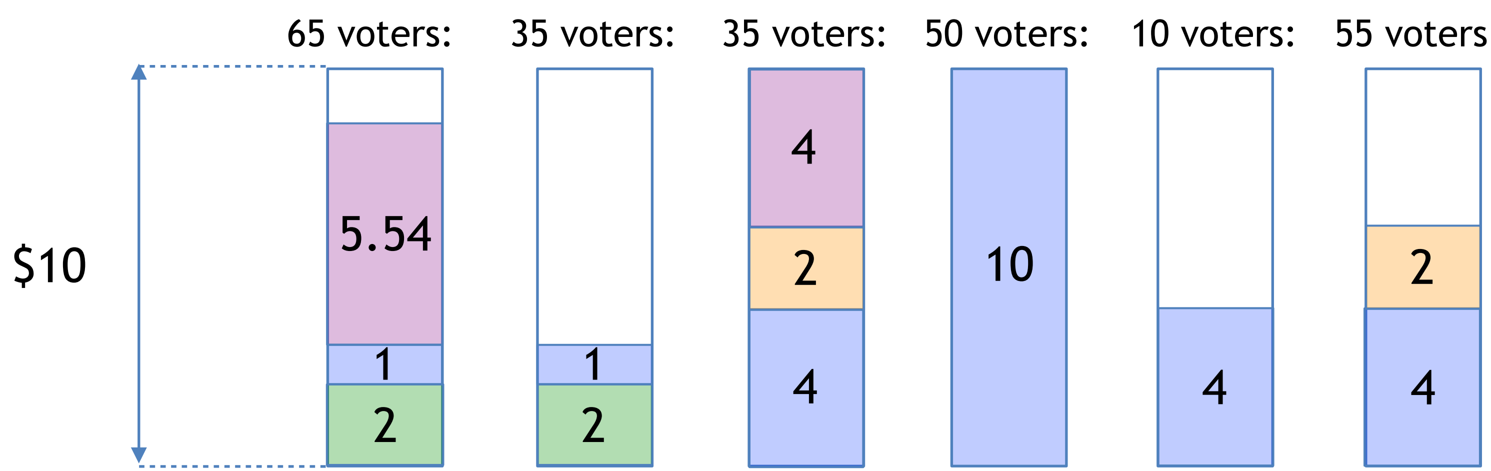

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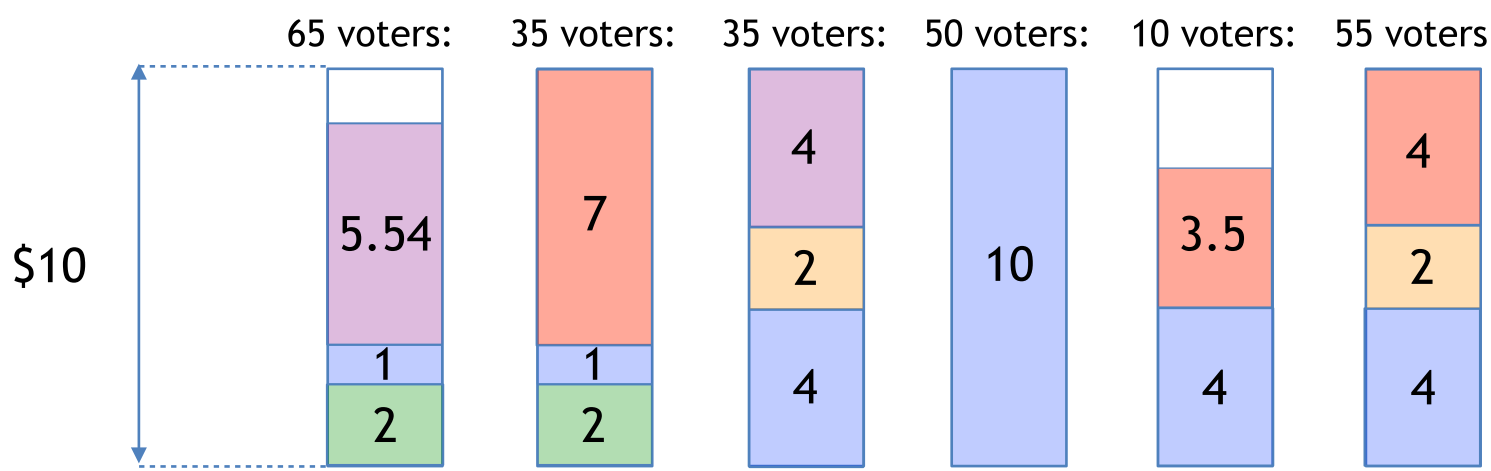

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Theorem: Method of equal shares satisfies extended justified representation up-to-one.

Can we get EJR (without up-to-one)?

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Theorem: There exists no polynomial-time algorithm that satisfies EJR.

Proof: For one voter this is simply the knapsack problem which is NP-hard.

Knapsack problem:
We are given a set of items, each with a weight and a value, and two integers: B, K.
Determine whether there exists a subset of items with total weight not exceeding B and with the total value at least equal to K.

How to use MES with approval ballots?

Given approval ballots we need to decide what is the utility?

There are two main choices:

1. The utility of a voter is the total amount of money spent on approved projects:

   $u_i(c) = \mathrm{cost}(c)$ if i approves c, and $u_i(c) = 0$, otherwise.

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Greedy Algorithm:
Select candidates with the highest ratio of value to the weight.

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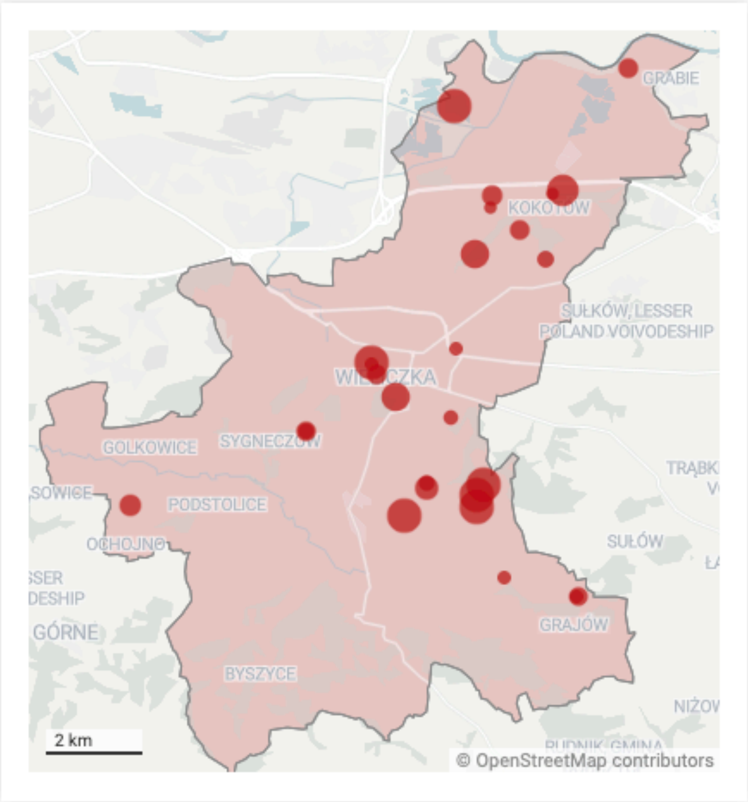
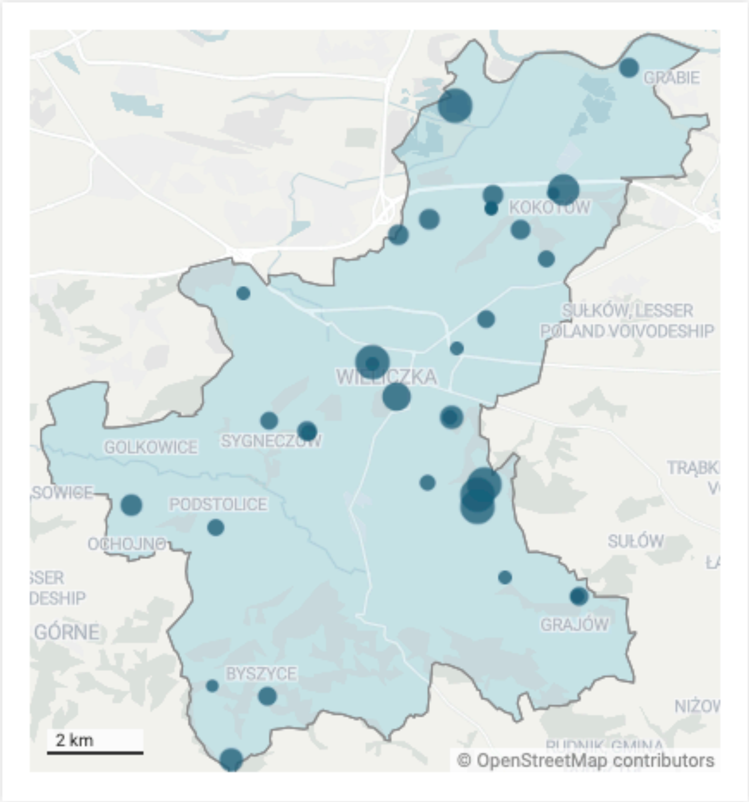
Such project maximises the value divided by the cost, where the value is the sum of utilities that the voters enjoy from the project, assuming the utility is defined using approach 1.

Example of usage in Wieliczka



standard method



method of equal shares

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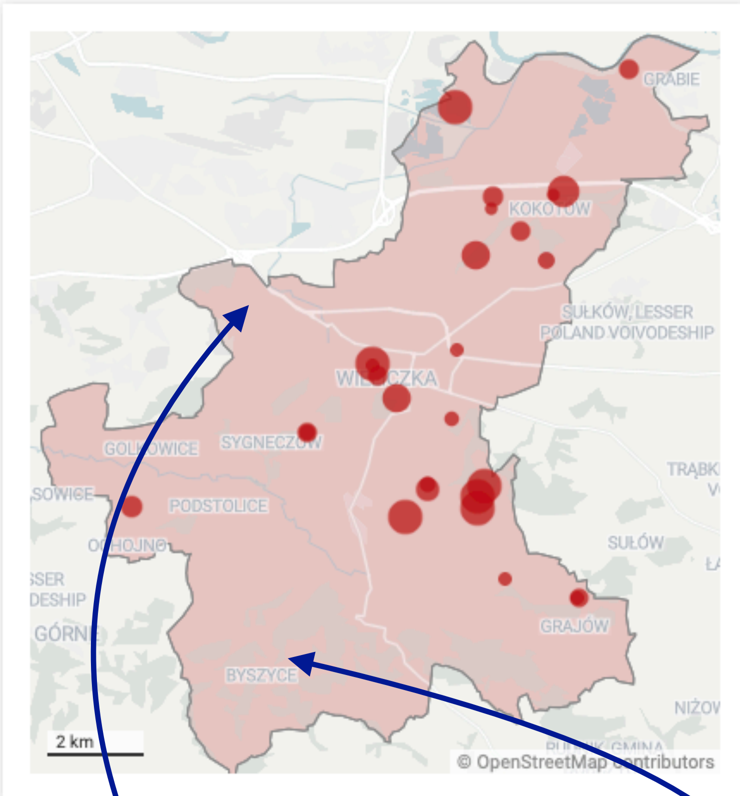

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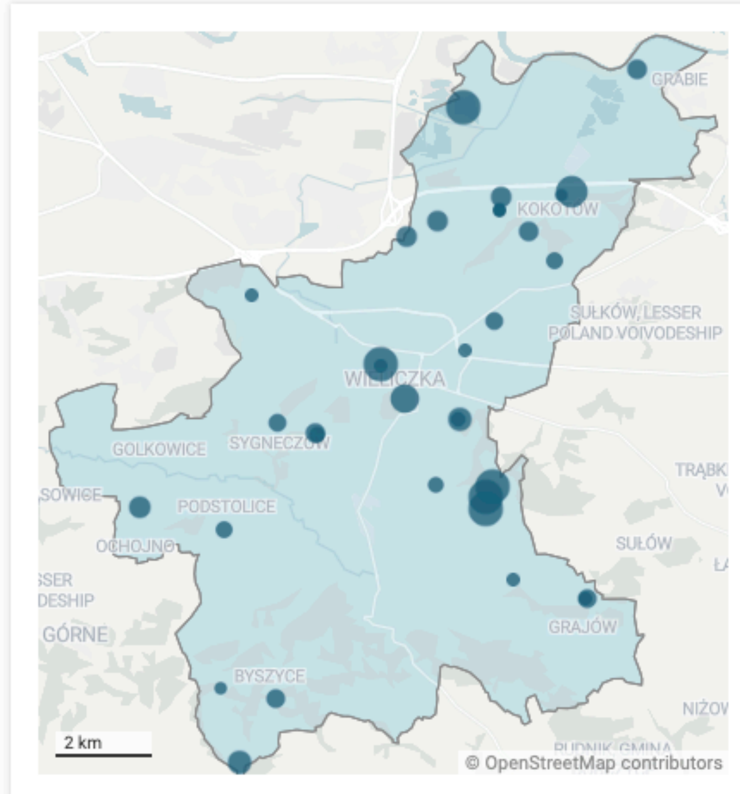

method of equal shares

In the standard majoritarian method, we would have discriminated regions

(We avoided this thanks to the method of equal shares)

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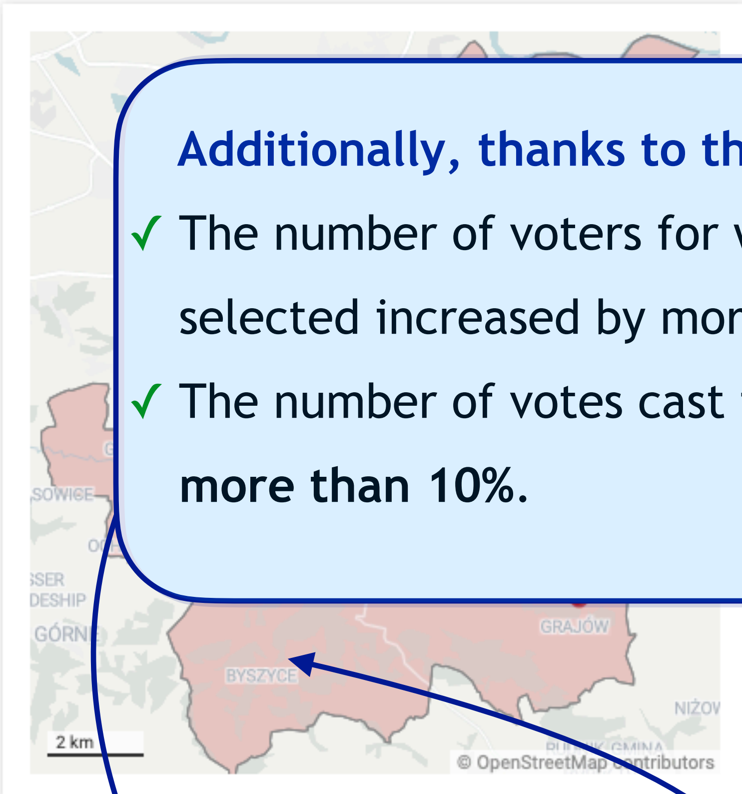

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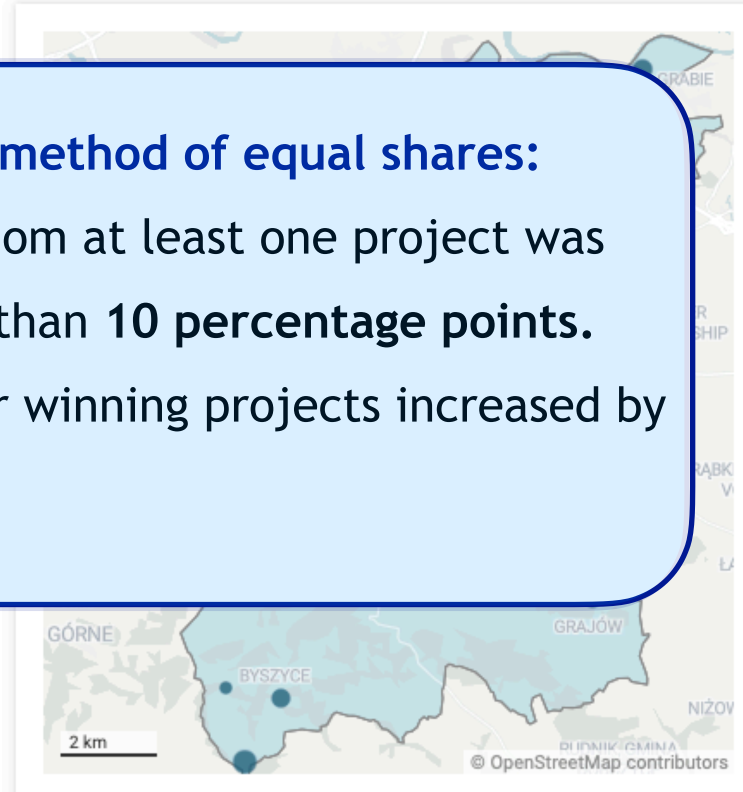

method of equal shares

Additionally, thanks to the method of equal shares:

- ✓ The number of voters for whom at least one project was selected increased by more than **10 percentage points.**
- ✓ The number of votes cast for winning projects increased by **more than 10%.**

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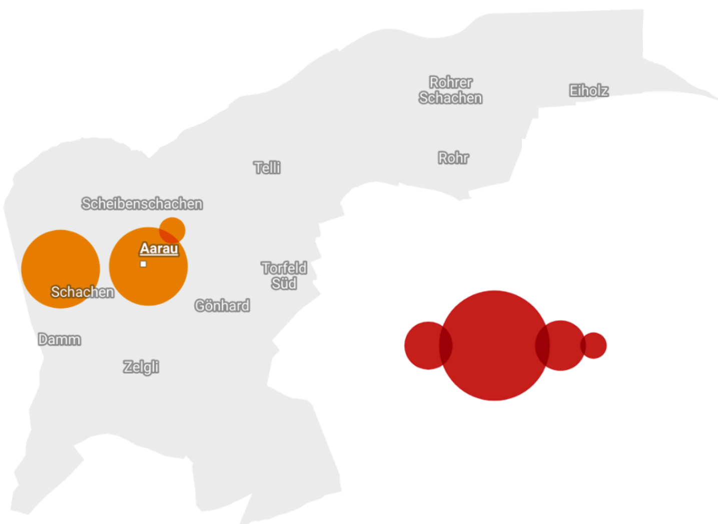
Example of usage in Aarau (Switzerland)



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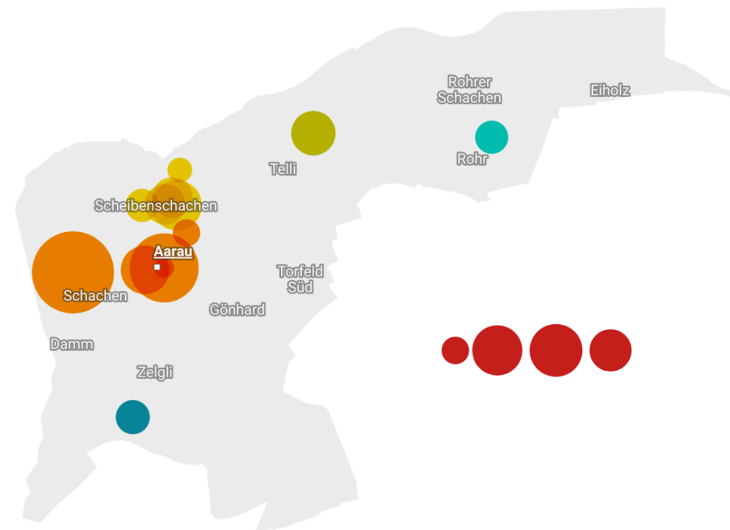

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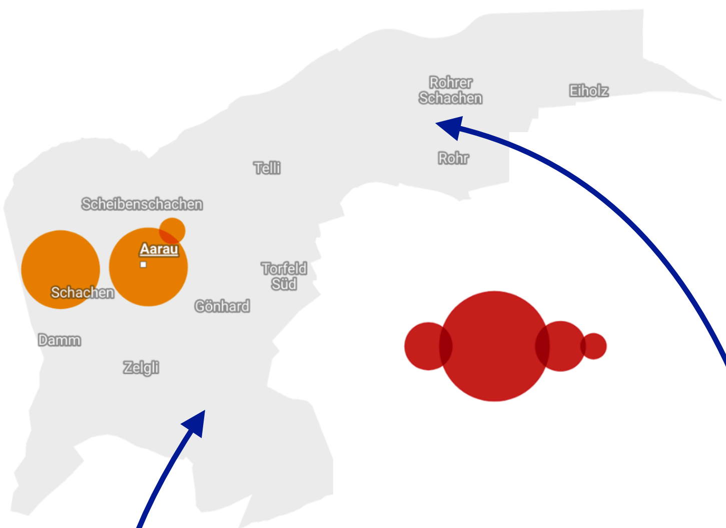

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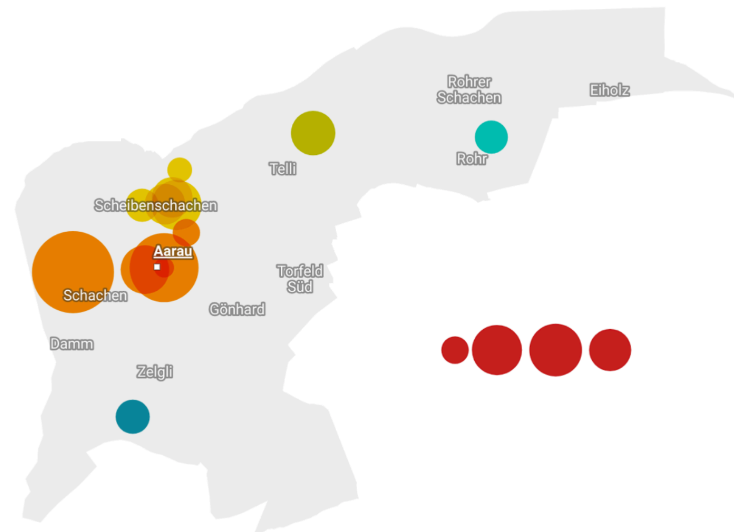

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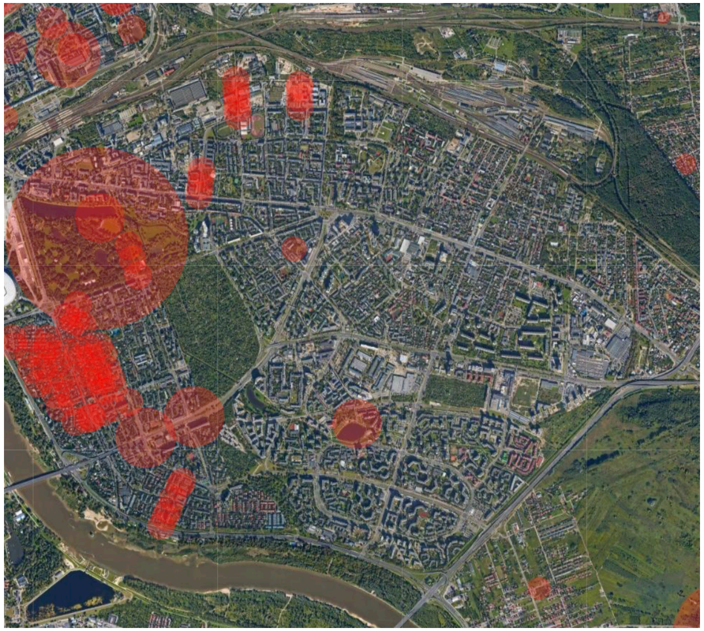
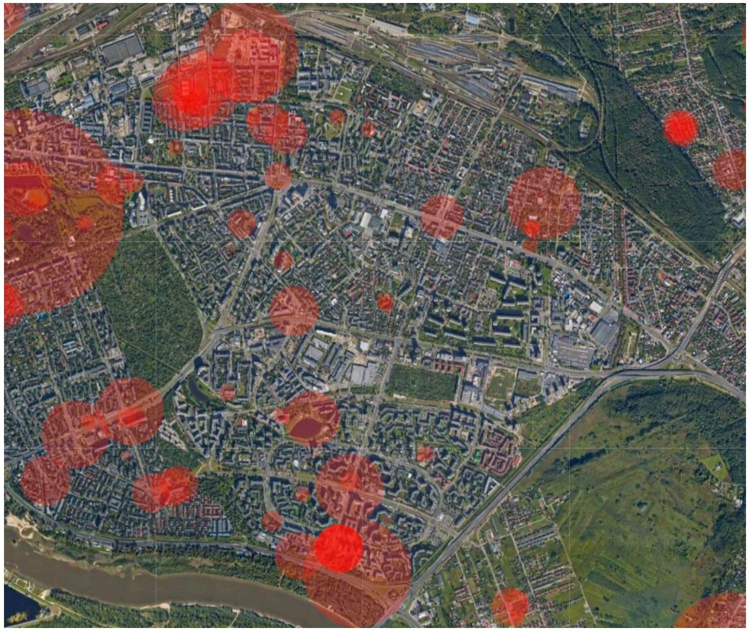
Geographical distribution of funds

(Warsaw, Praga District 2021)



standard method



method of equal shares

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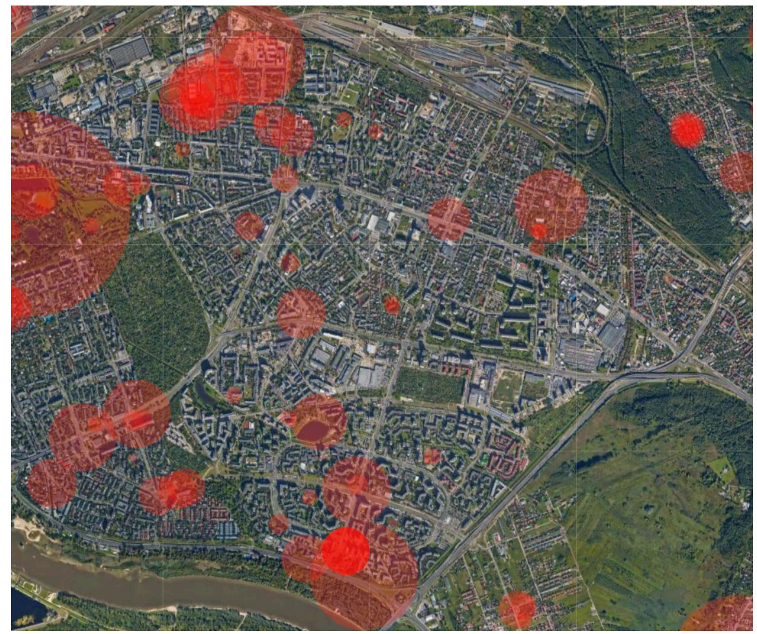

method of equal shares

discriminated region

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(Warsaw, Praga District 2021)

standard method

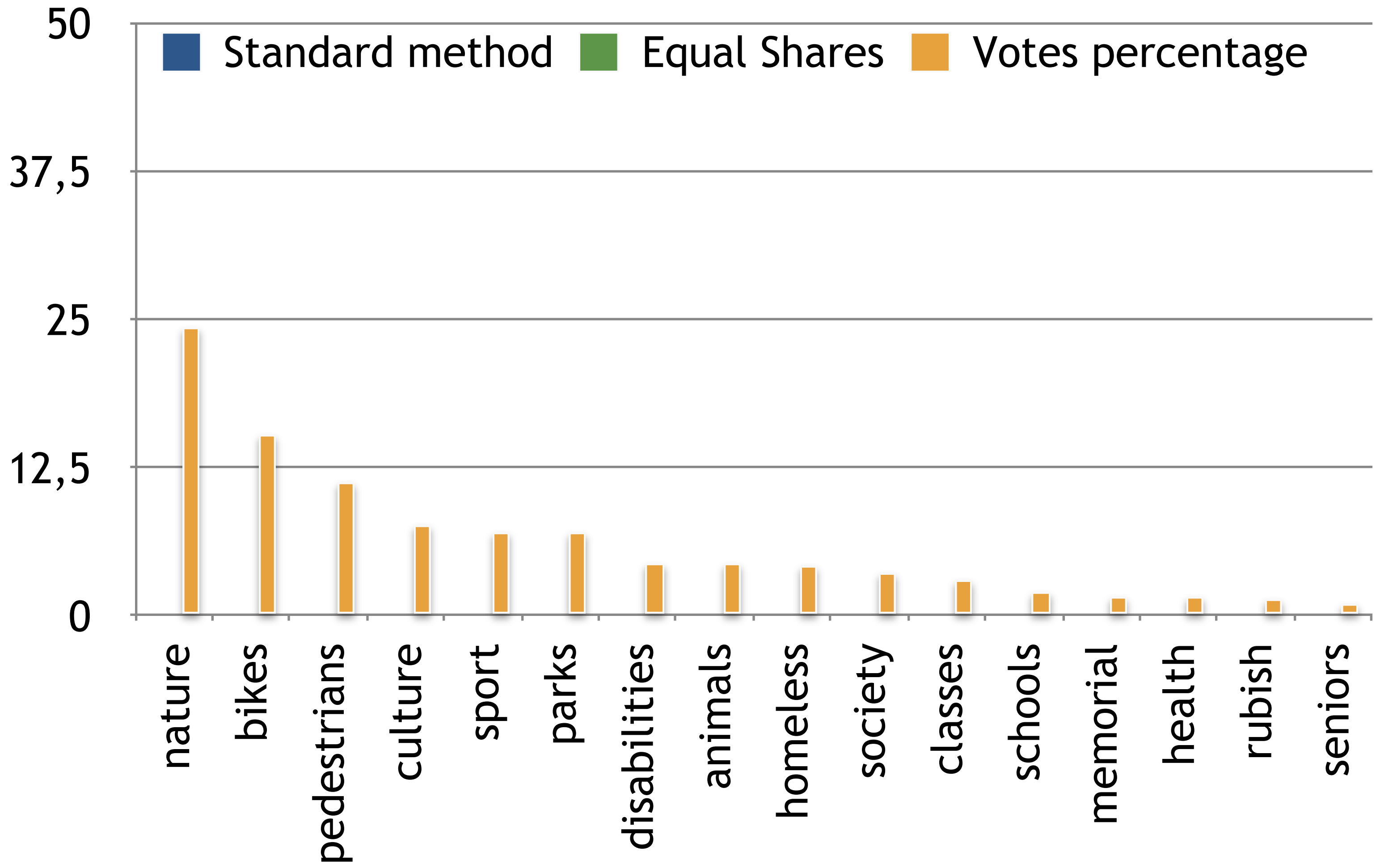
method of equal shares

discriminated region

the new method guarantees equal treatment

Distribution of funds among categories

(Warsaw 2022, citywide projects)



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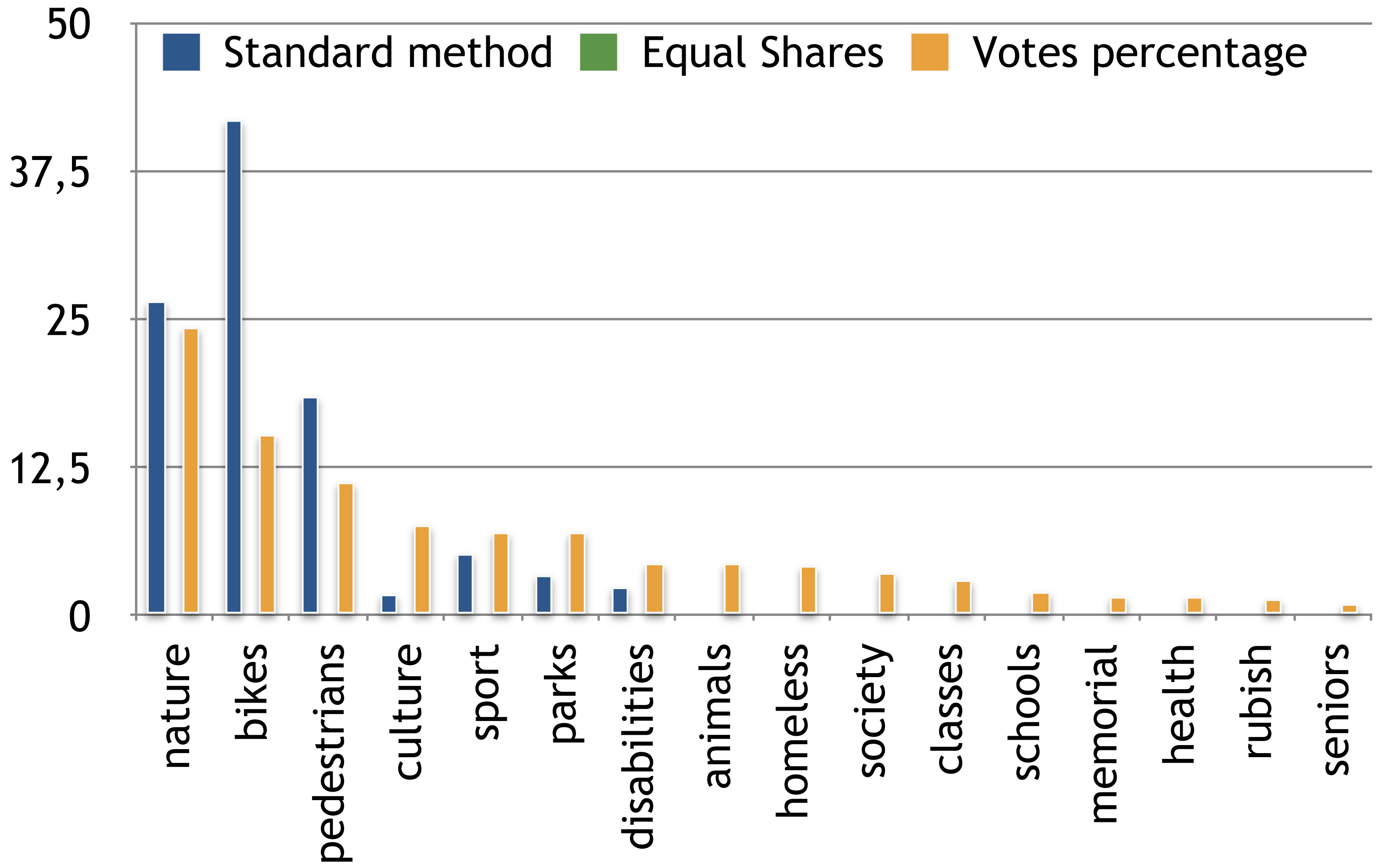

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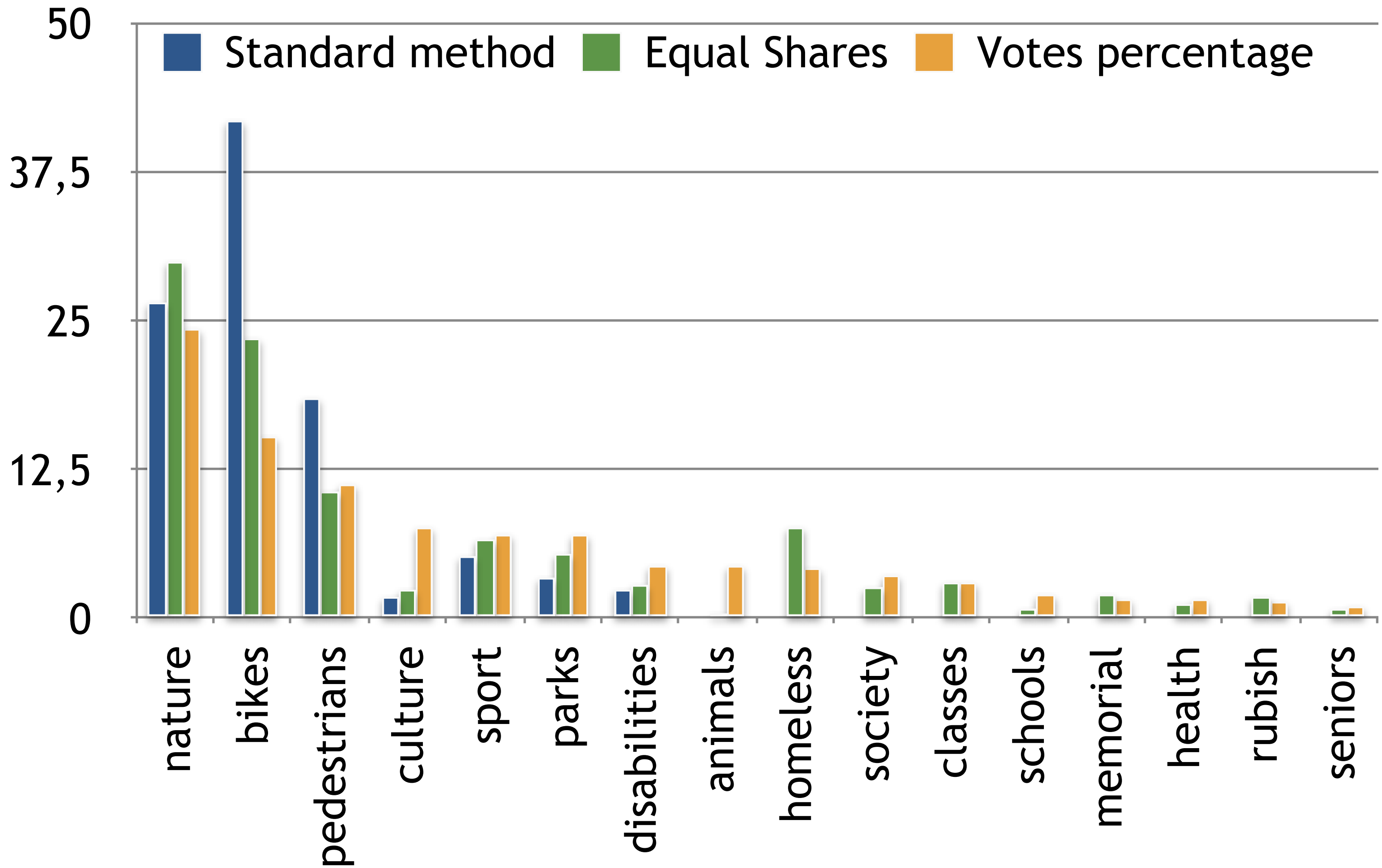


Resistance to strategies

CITIZENS' BUDGET FOR 2023

RECOMMENDED PROJECTS OF GENERAL NATURE TO BE IMPLEMENTED

No.	Name of the project task	Value of the project task [zł]
1.	Improving the level of safety in the localities of Mietniów, Pawlikowice, Chorągwica, Grajów, Dobranowice, Jankówka, Raciborsko, Lednica Górna, Podstolice, Gorzów, Janowice	498,033.00
2.	Improving living conditions and safety for residents of the villages: Brzegi, Byszyce, Czarnocowice, Grabie, Kokotów, Mała Wieś, Strumiany, Sułków, Śledziejowice, Węgrzce Wielkie, Zabawa	500,000.00

Resistance to strategies

CITIZENS' BUDGET FOR 2023

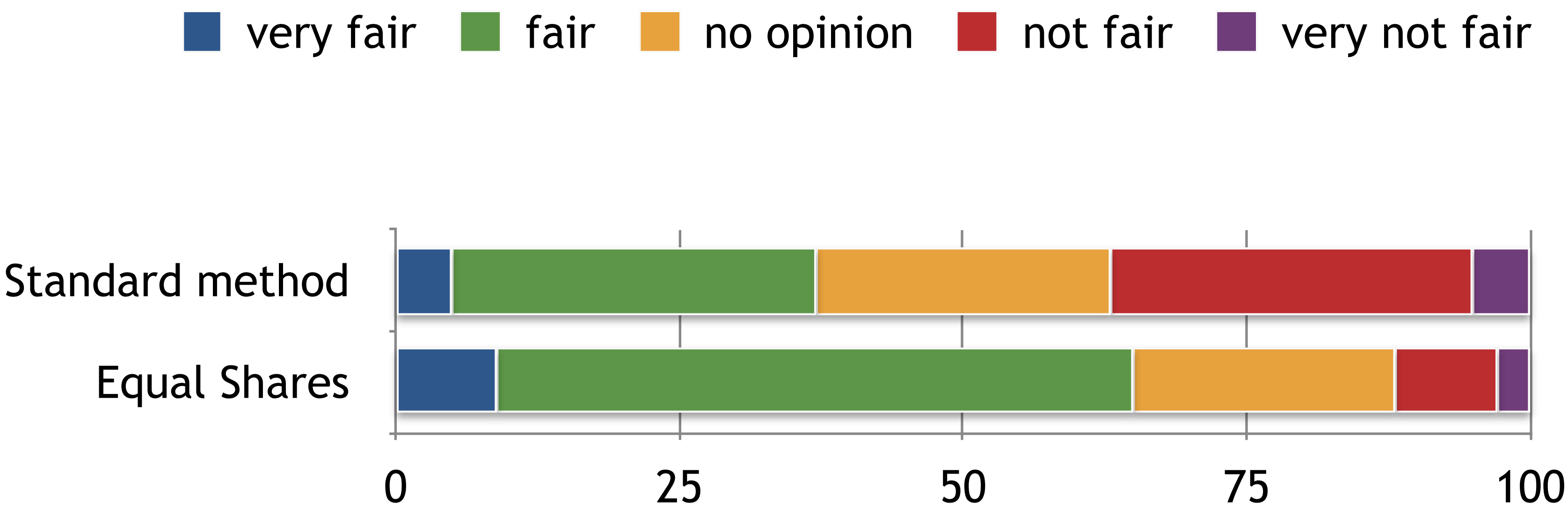
RECOMMENDED PROJECTS OF GENERAL NATURE TO BE IMPLEMENTED

No.	Name of the project task	Value of the project task [zł]
1.	Improving the level of safety in the localities of Mietniów, Pawlikowice, Chorągwica, Grajów, Dobranowice, Jankówka, Raciborsko, Lednica Górna, Podstolice, Gorzów, Janowice	498,033.00
2.	Improving living conditions and safety for residents of the villages: Brzegi, Byszyce, Czarnocowice, Grabie, Kokotów, Mała Wieś, Strumiany, Sułków, Śledziejowice, Węgrzce Wielkie, Zabawa	500,000.00

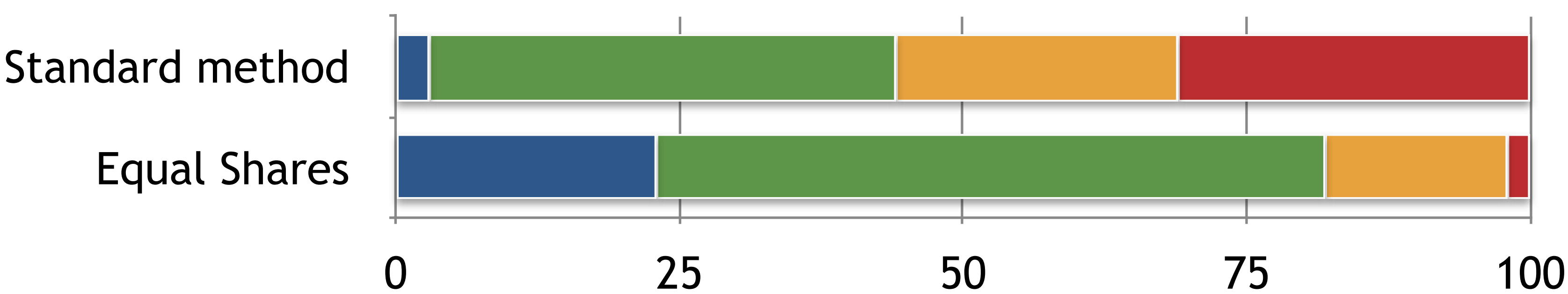
In the old method, project proposers use strategies to eliminate competition.

(Golkowice, Grabówki, Koźmice Wielkie, Koźmice Małe, Rożnowa, Siercza, Sygneczów, Wieliczka Miasto did not receive any project)

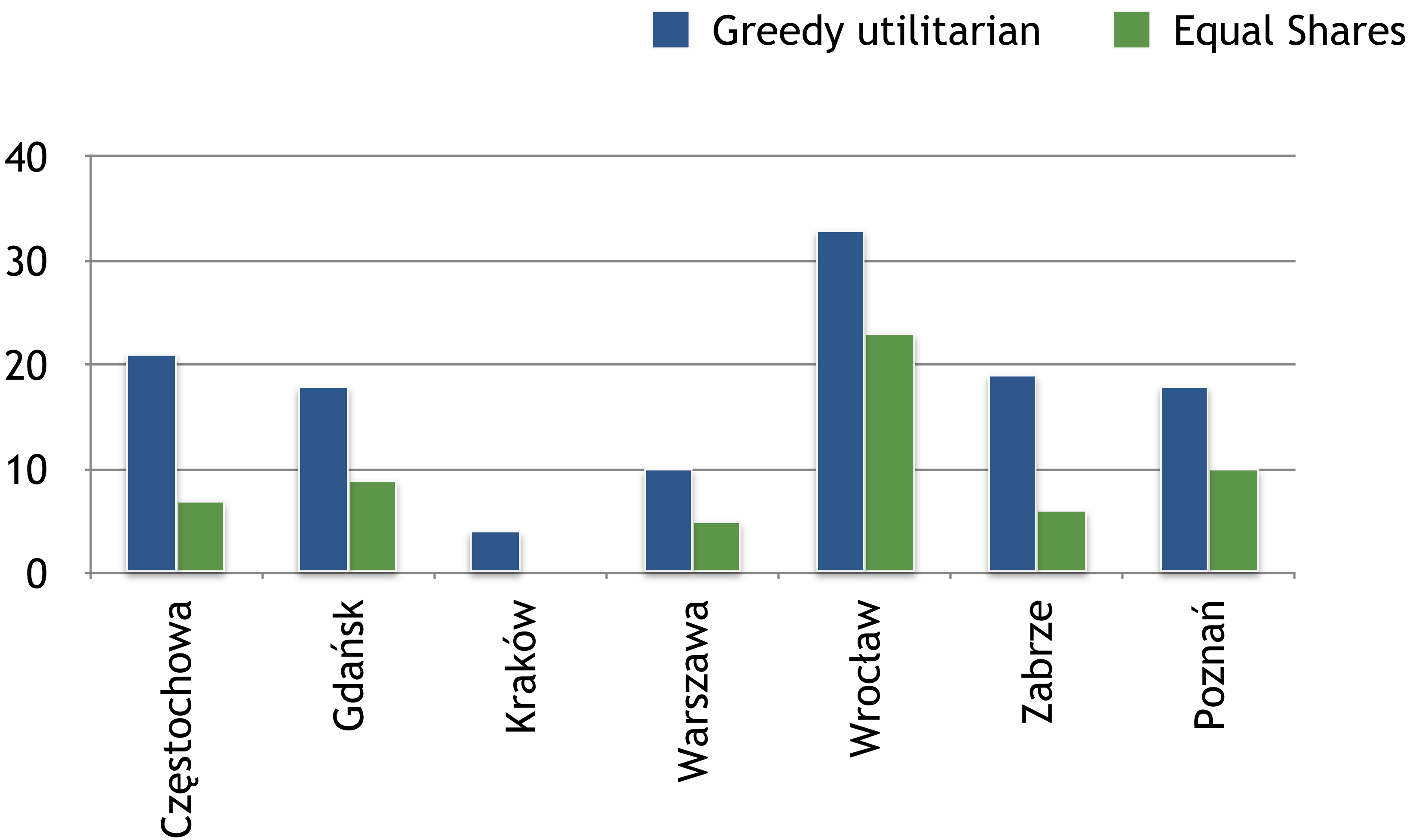
Surveys conducted in Switzerland



After showing the explanation:

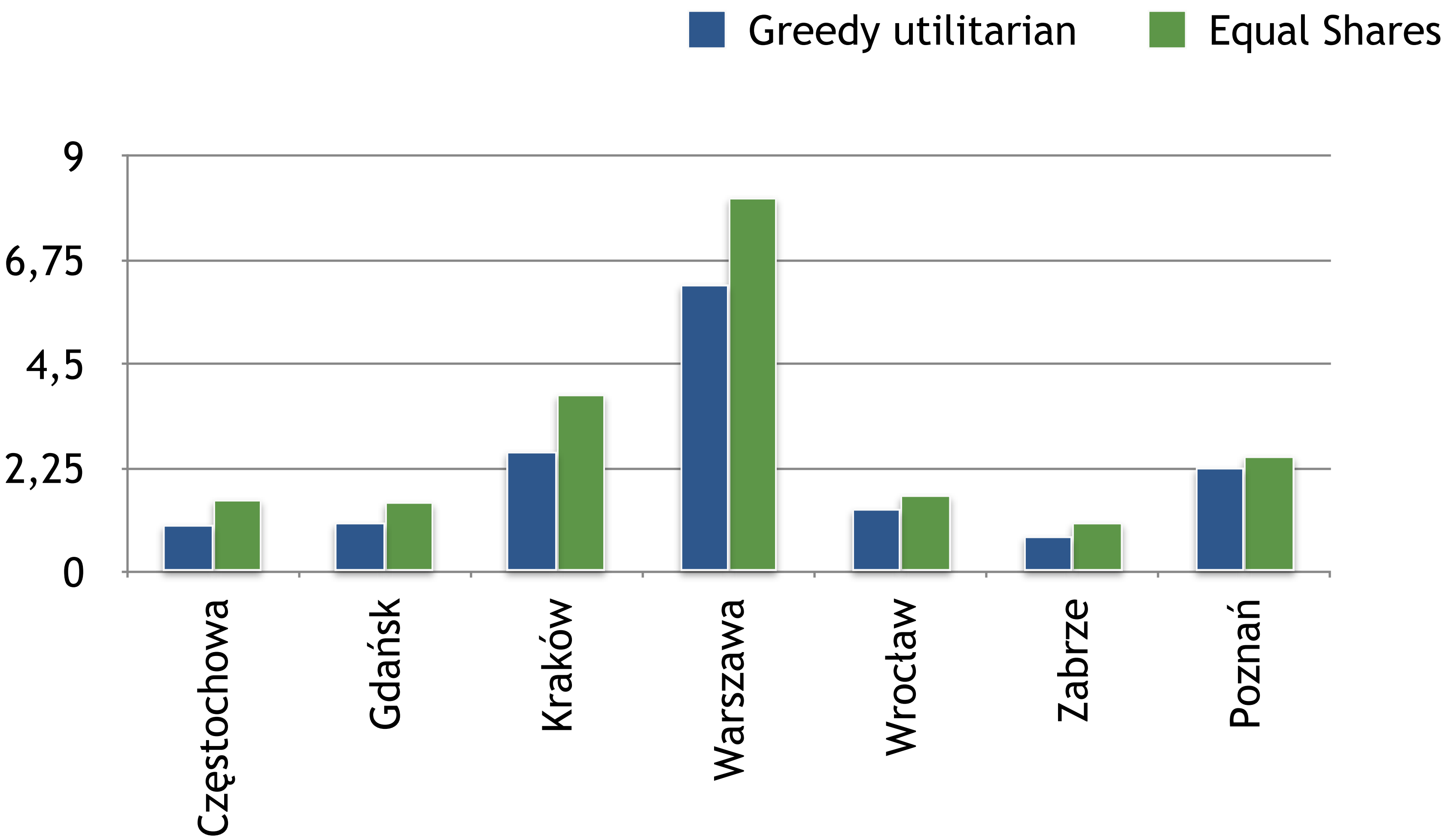


Percentage of voters with no project



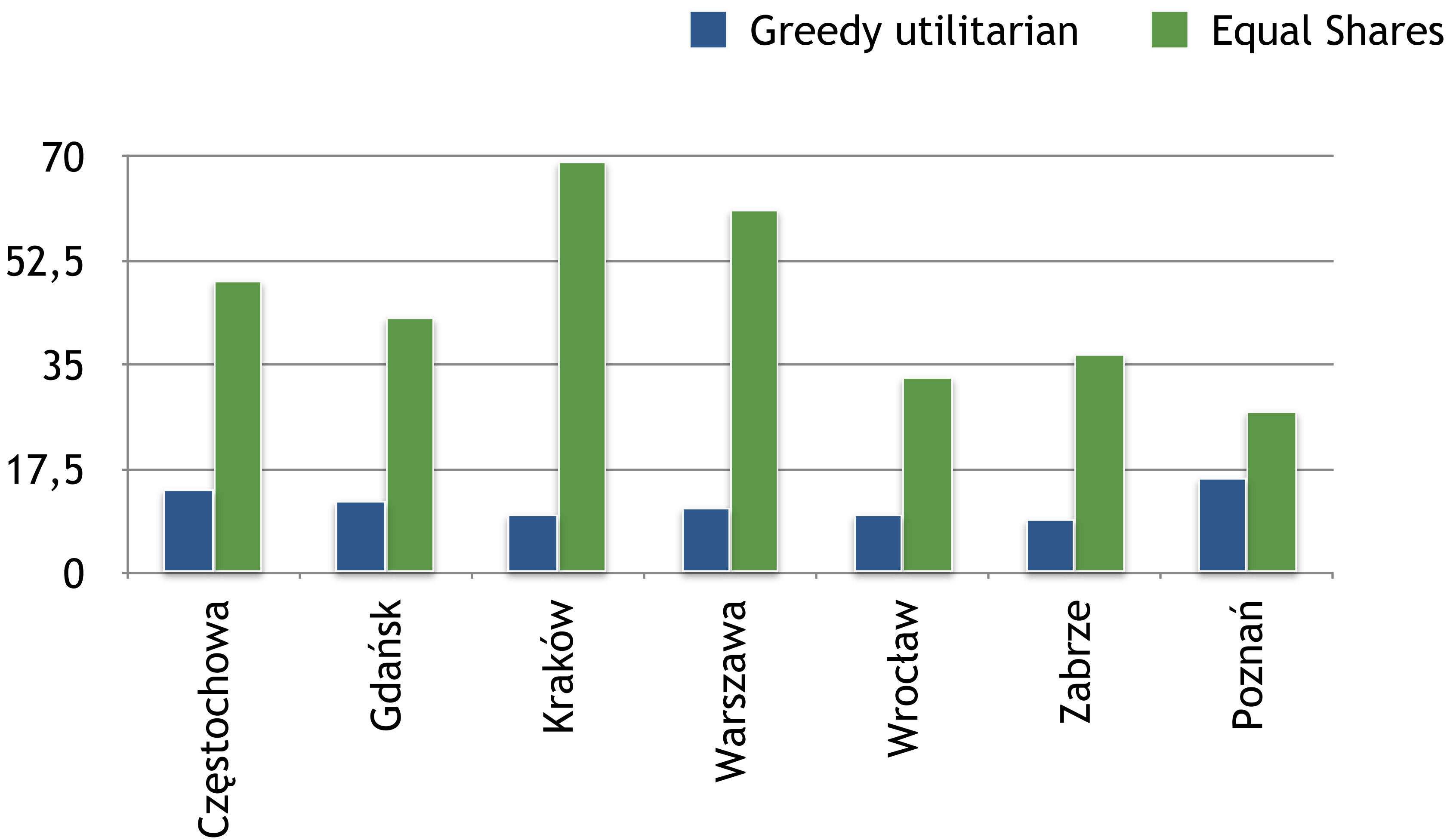
Average voter satisfaction

(Number of approved projects)



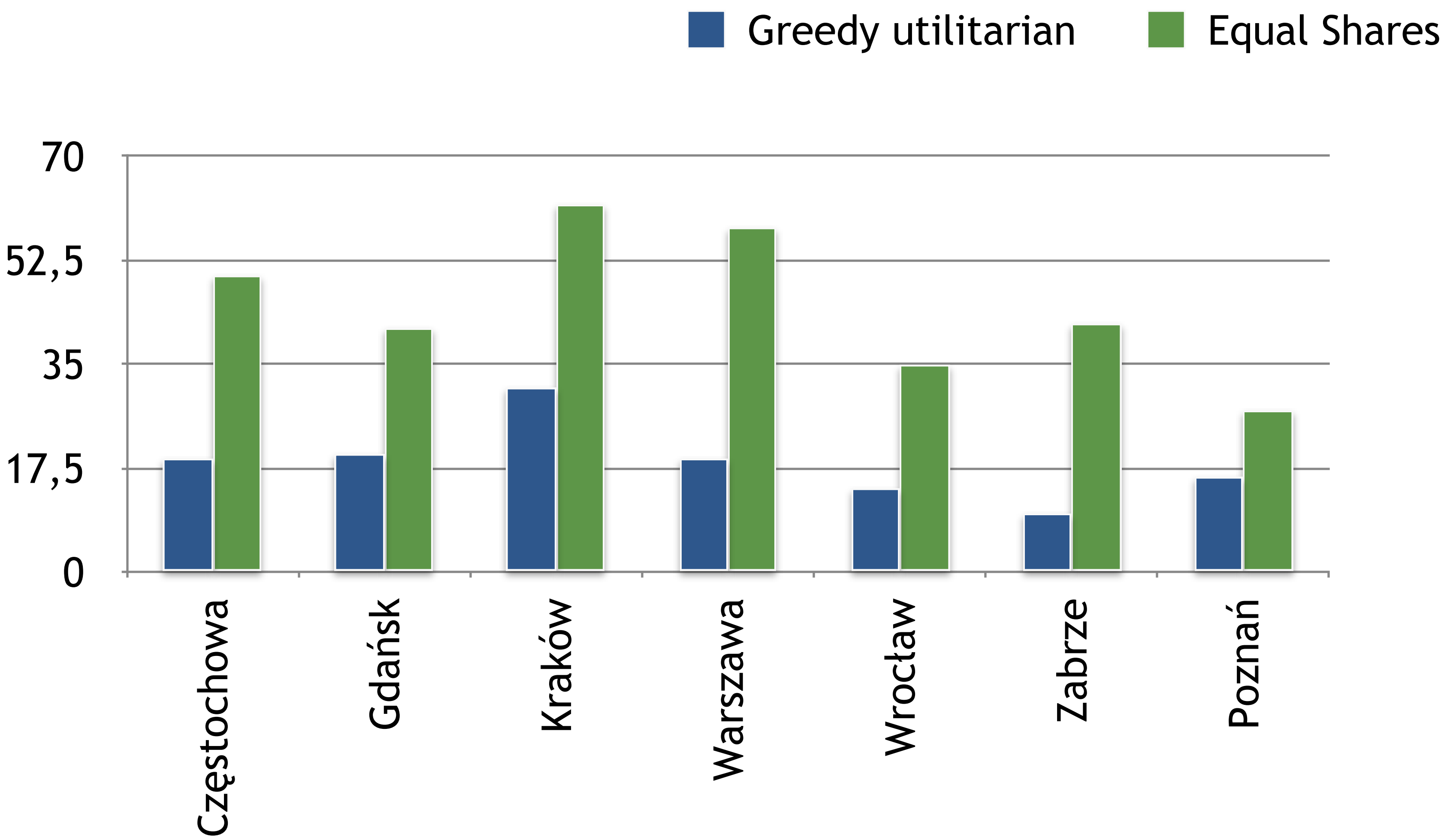
Average voter satisfaction

(Total cost of approved projects)



Percentage of voters with higher satisfaction

(Total cost of approved projects)



Conclusion

- ✓ **Better reflects** voters' preferences.
- ✓ Leads to **higher voter satisfaction**.
- ✓ Respondents consider it **fairer and more trustworthy**.
- ✓ **The voting process remains the same**.

Method of Equal Shares

More:

https://equalshares.net/