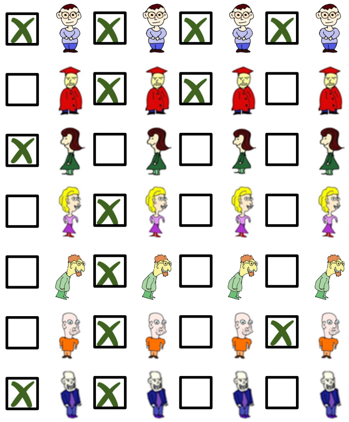
Proportional Algorithms:

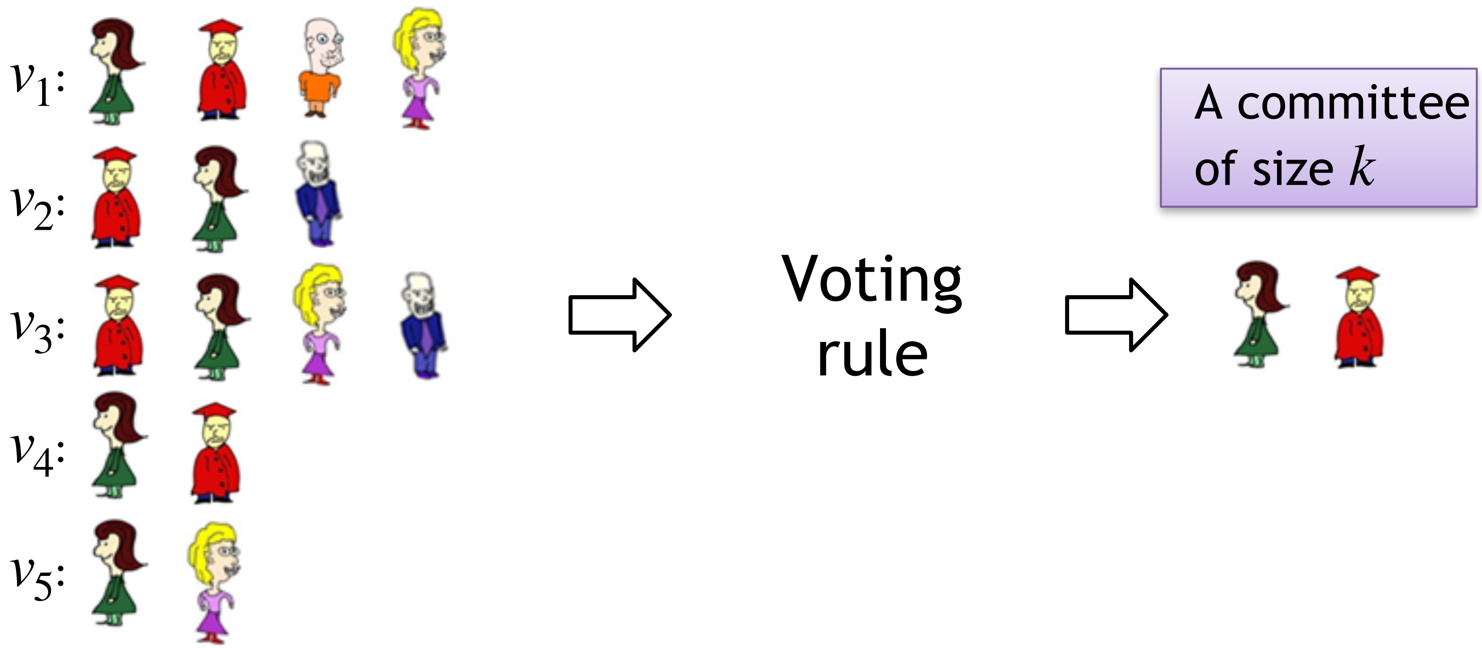
Approval-Based Committee Elections



Piotr Skowron

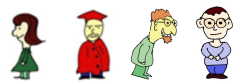
University of Warsaw

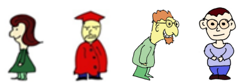
Model: Approval-Based Elections

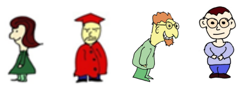


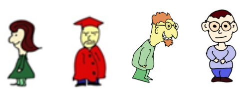
A preference profile: an example

We have $n = 8$ voters, $m = 9$ candidates.

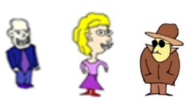
v_1: 

v_2: 

v_3: 

v_4: 

v_5: 

v_6: 

v_7: 

v_8: 

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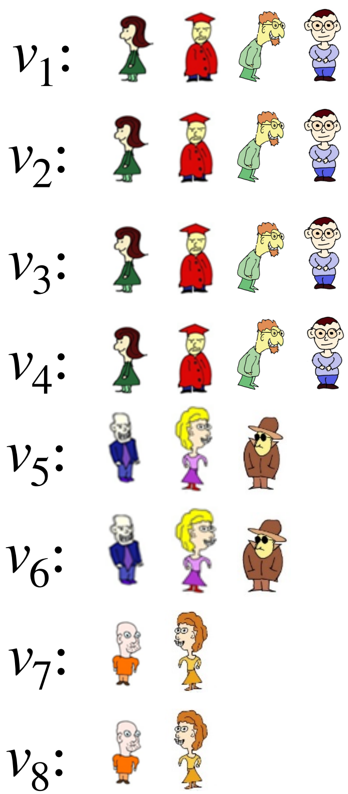

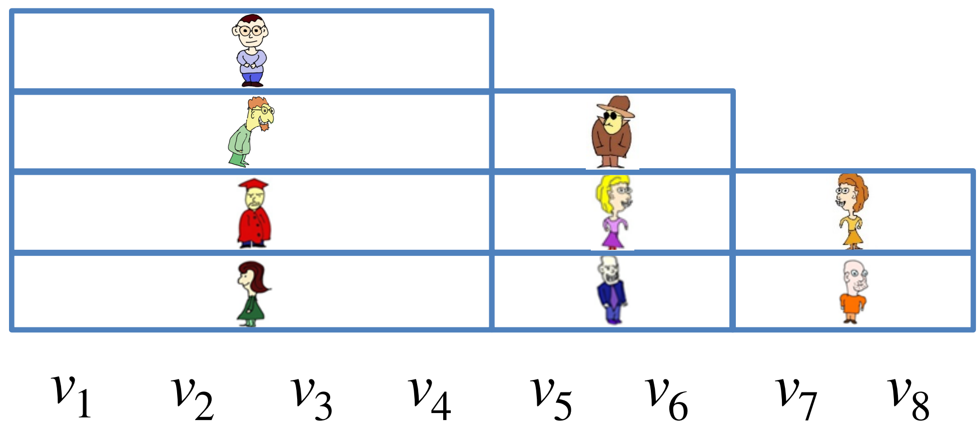

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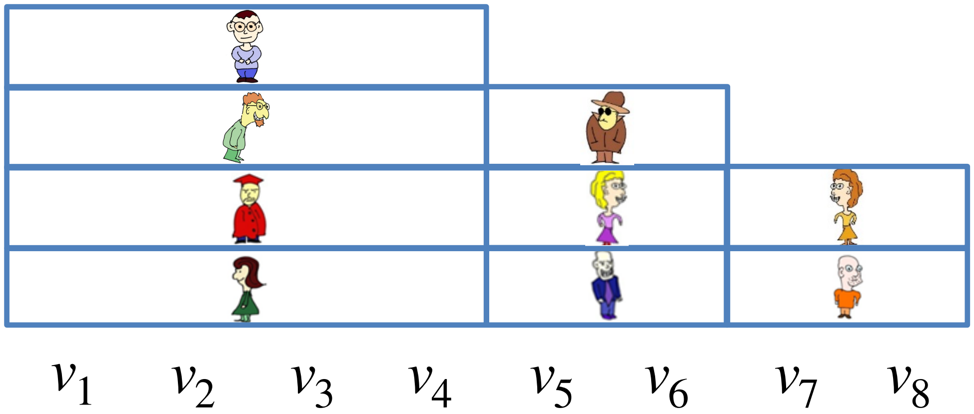

Assume the committee size to be elected is $k = 4$.

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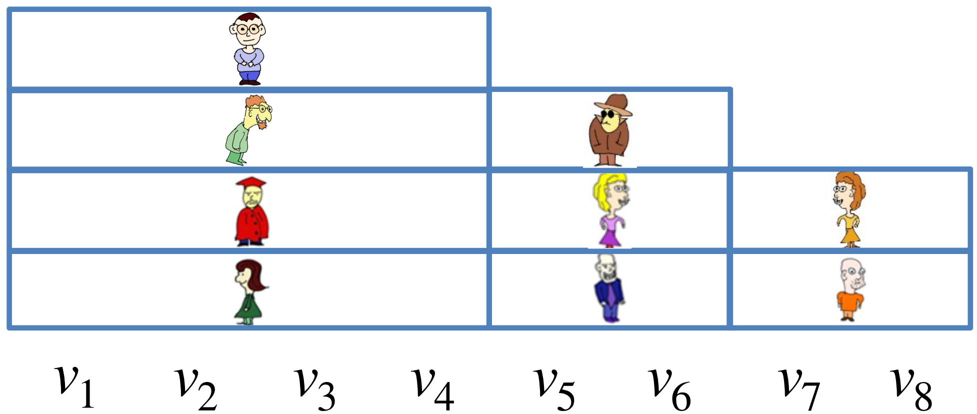

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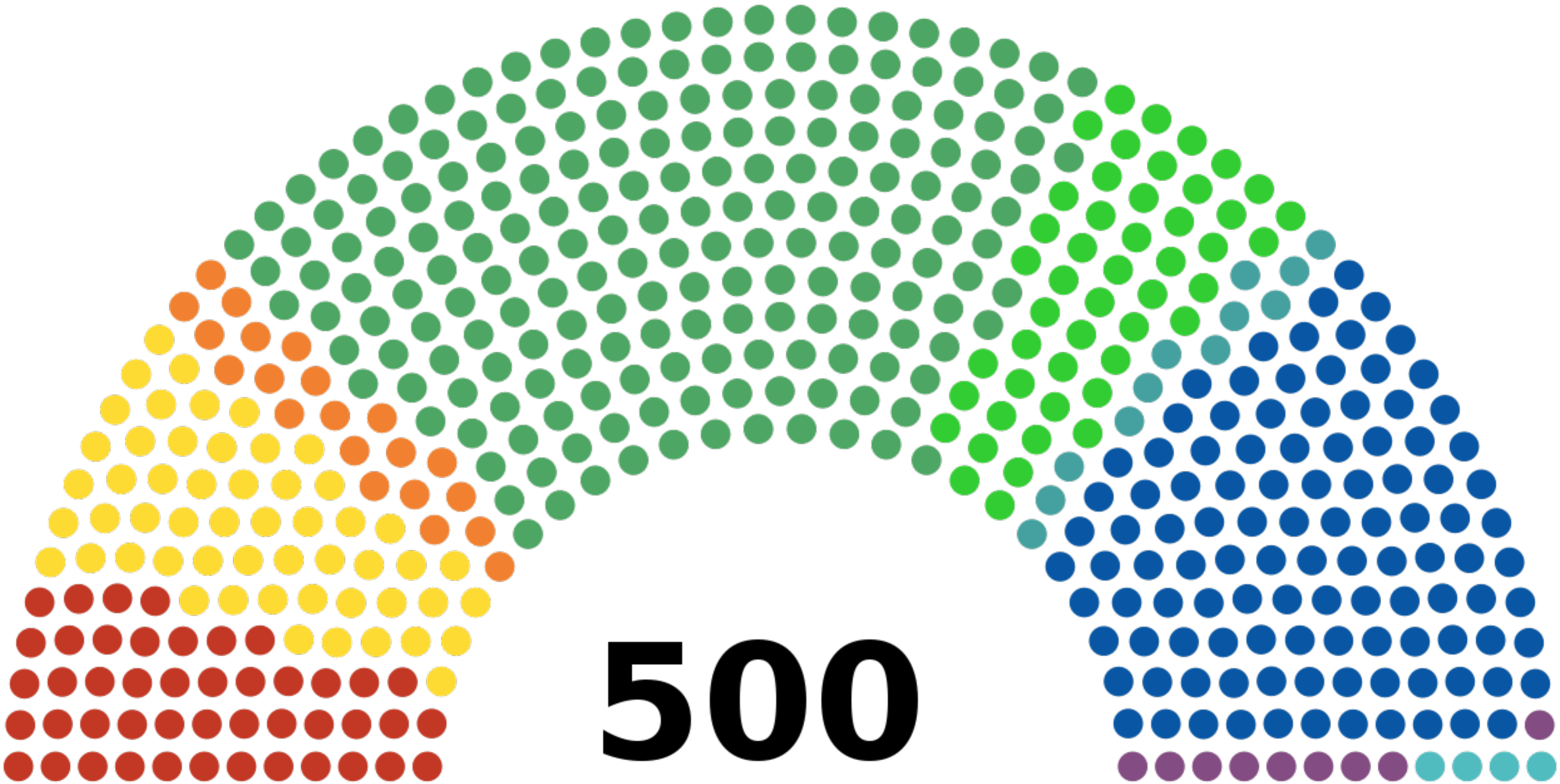
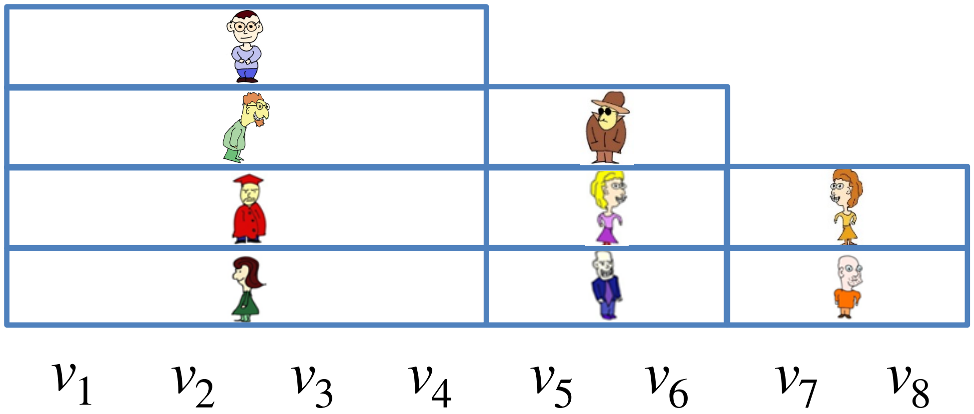
Which committee should be selected?

Context: electing a representative body



Back to the example!



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In this context the committee should be proportional.

But what does it mean and how could we achieve that?

Proportionality on the example of party-list systems.

Each voter casts one vote for a single party.

Our goal is to select a committee of size $k = 4$:

- **Party 1** gets **40** votes.
- **Party 2** gets **20** votes.
- **Party 3** gets **20** votes.

How should the parliament look like?

Proportionality on the example of party-list systems.

Each voter casts one vote for a single party.

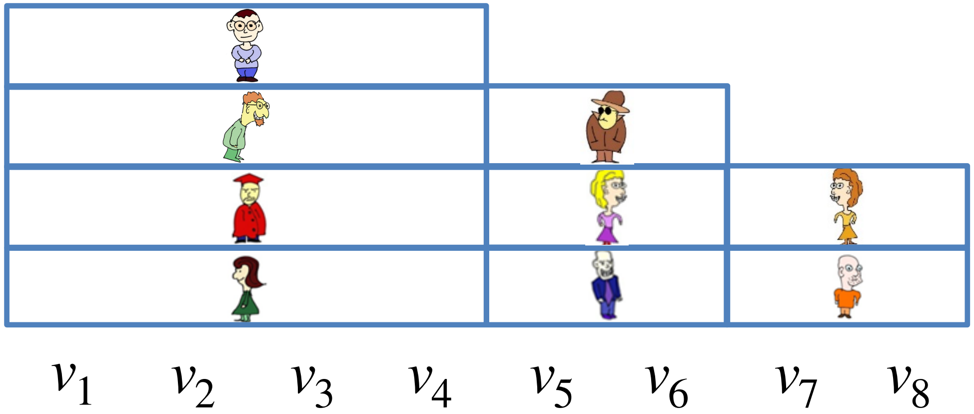
Our goal is to select a committee of size $k = 4$:

- Party 1 gets 40 votes.
- Party 2 gets 20 votes.
- Party 3 gets 20 votes.

How should the parliament look like?

- Party 1 should get 2 seats.
- Party 2 should get 1 seat.
- Party 3 should get 1 seat.

Back to the example!



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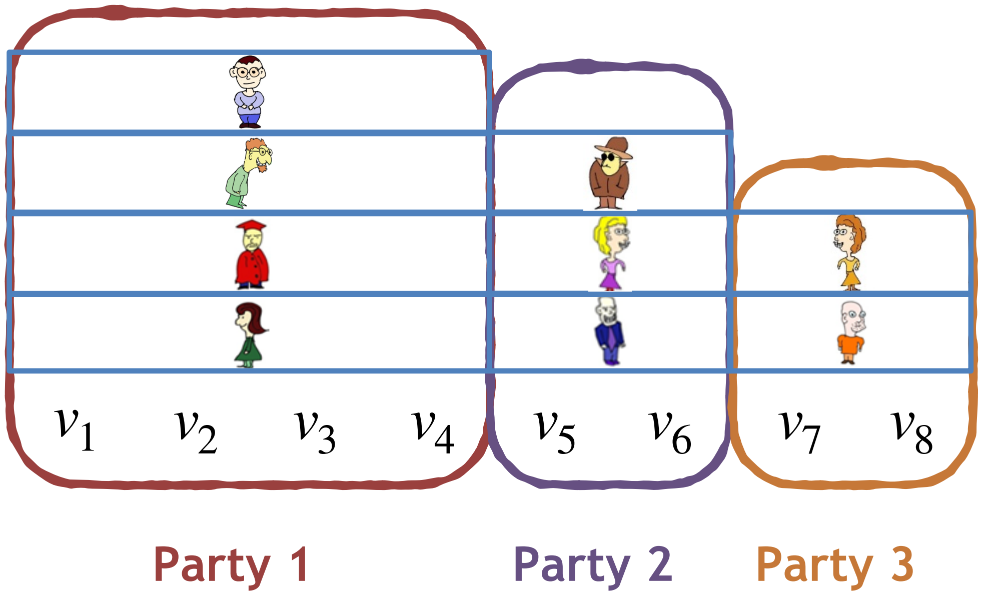

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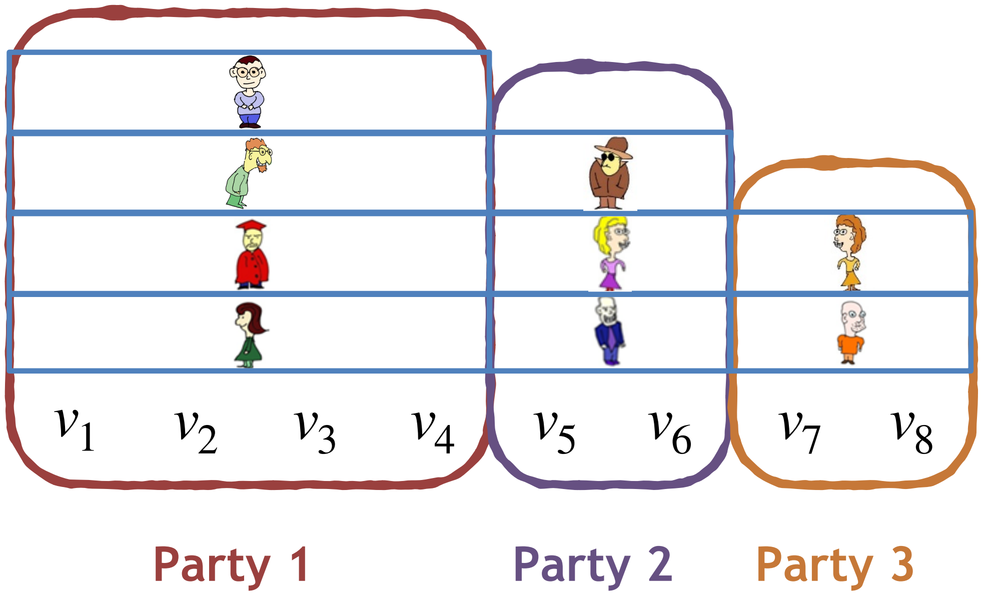

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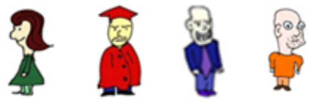
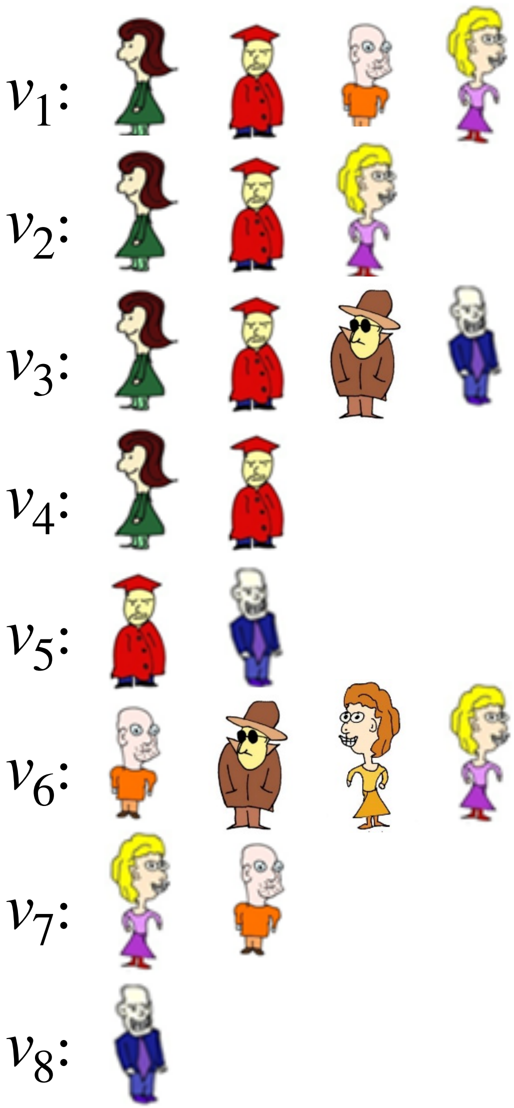


How to define proportionality for more complex preferences?



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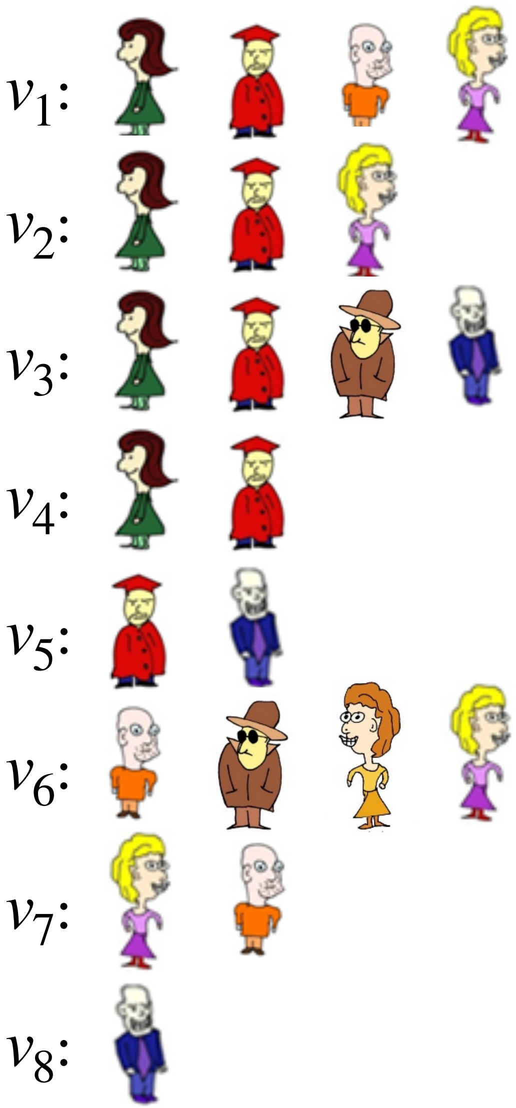

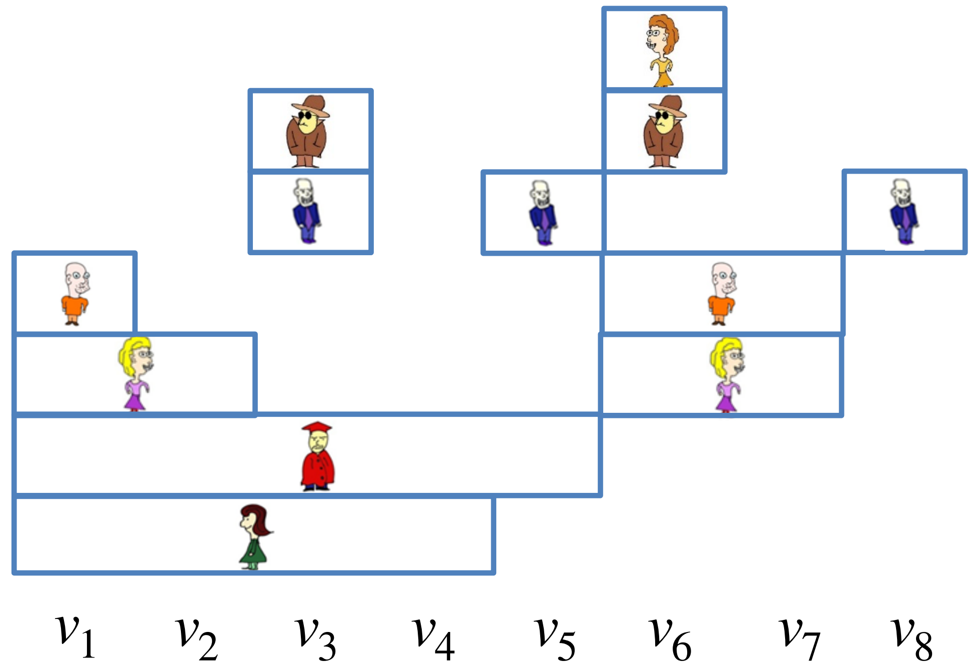

Let's move back in time to the end of the 19th century?

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Thorvald N. Thiele

Edvard Phragmén

Proportional Approval Voting (Thiele)

Assume voter v approves t members of a committee W. Then v gives to W the following number of points:

$$\sum_{i=1}^{t} \frac{1}{i} = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{t}$$

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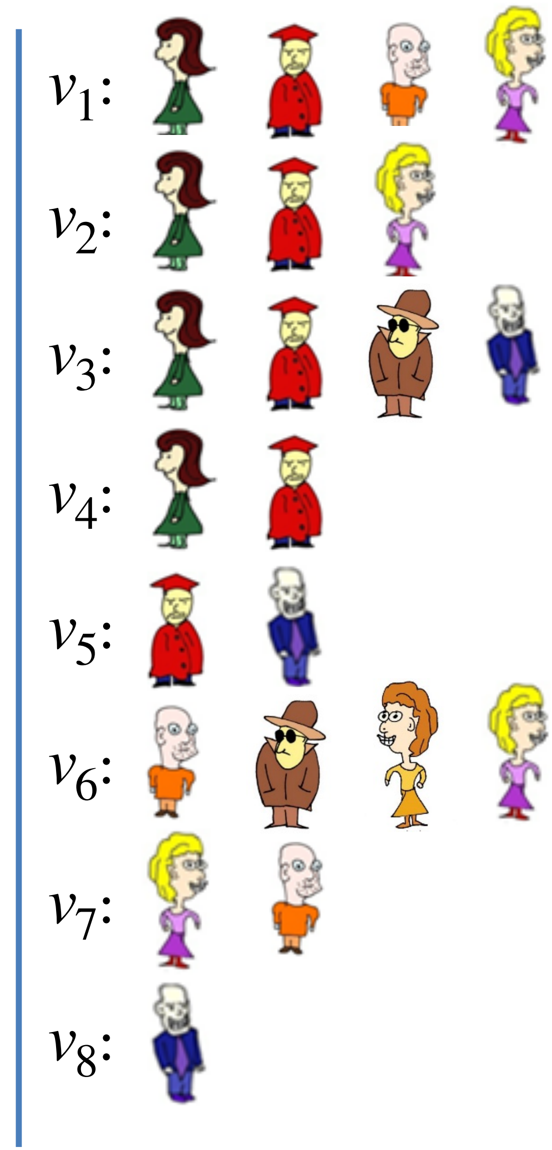

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E.g., consider a committee 

Points per voter:

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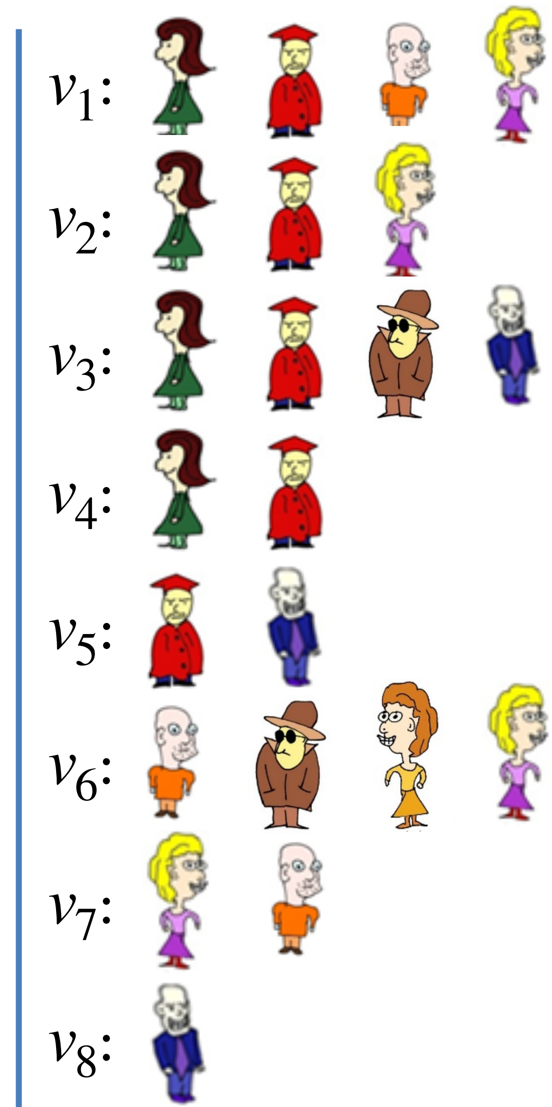

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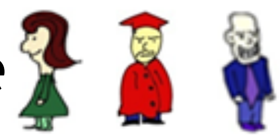
E.g., consider a committee 

Points per voter:

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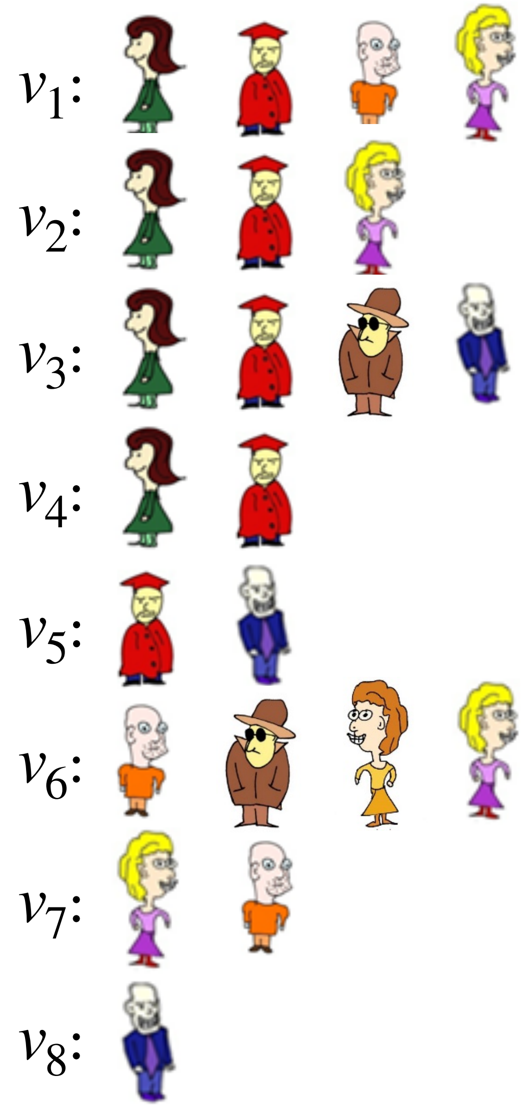

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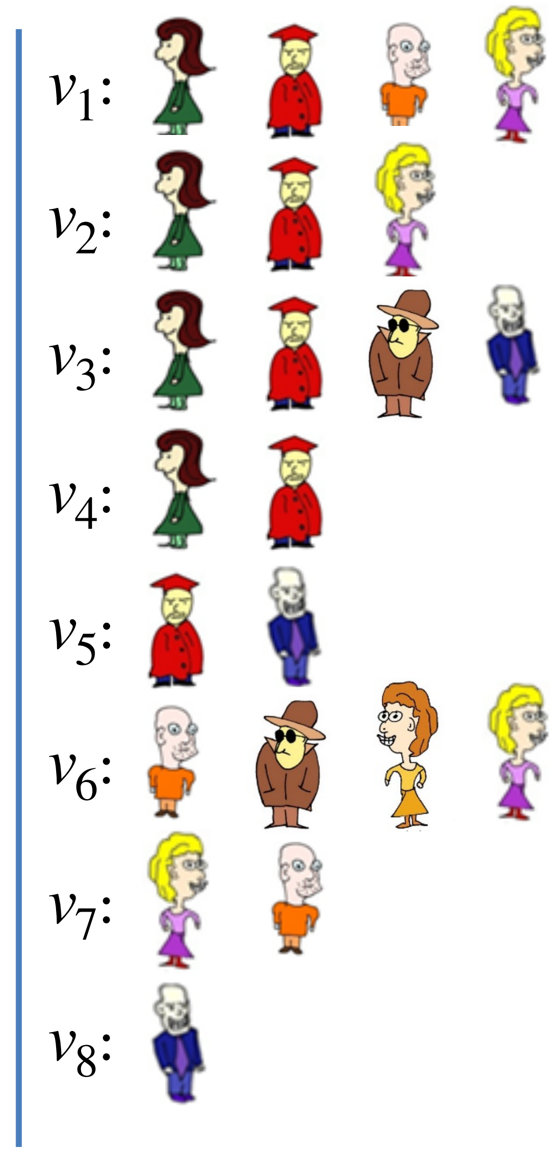

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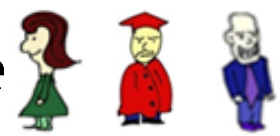
E.g., consider a committee 

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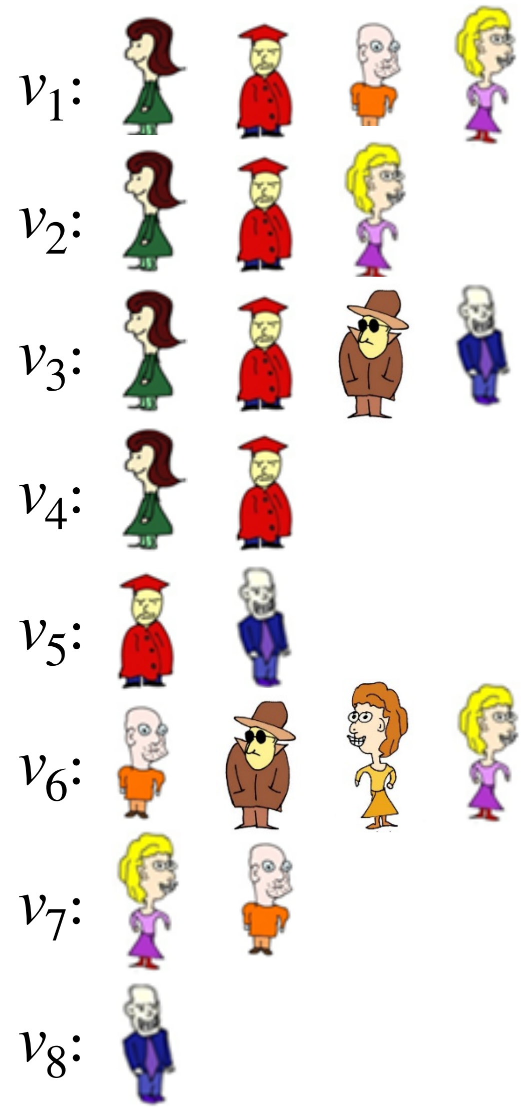

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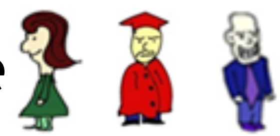
E.g., consider a committee 

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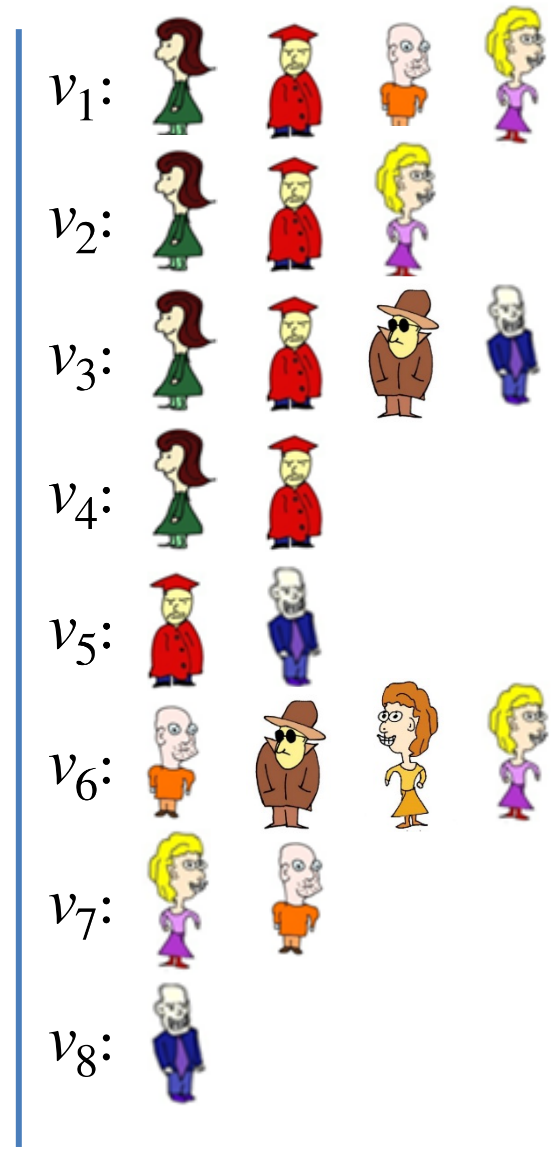

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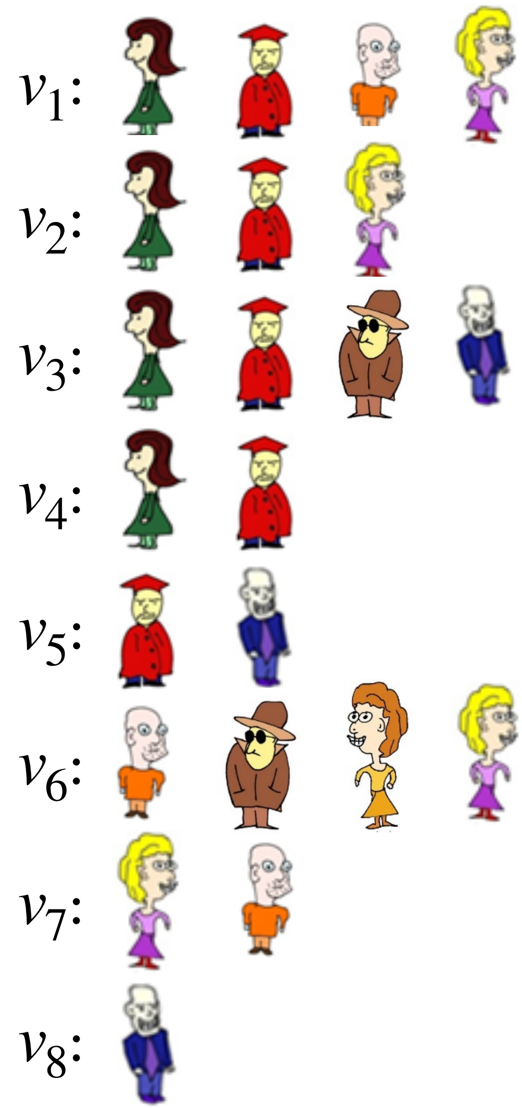

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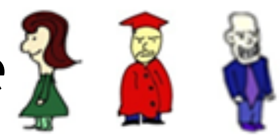
E.g., consider a committee 

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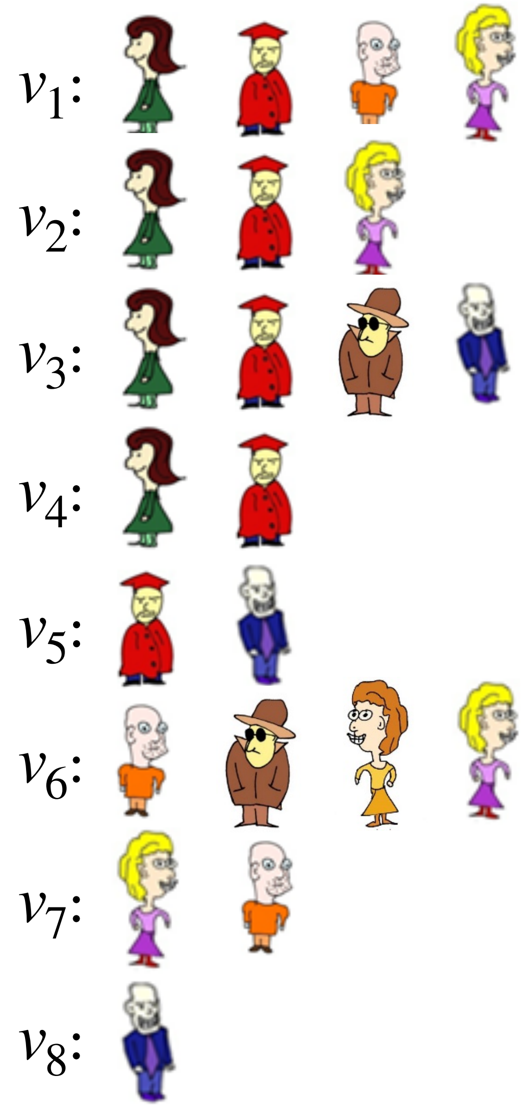
$v_5: 1 + \frac{1}{2}$ $\qquad$ $v_6: 0$



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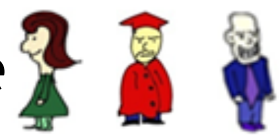
E.g., consider a committee 

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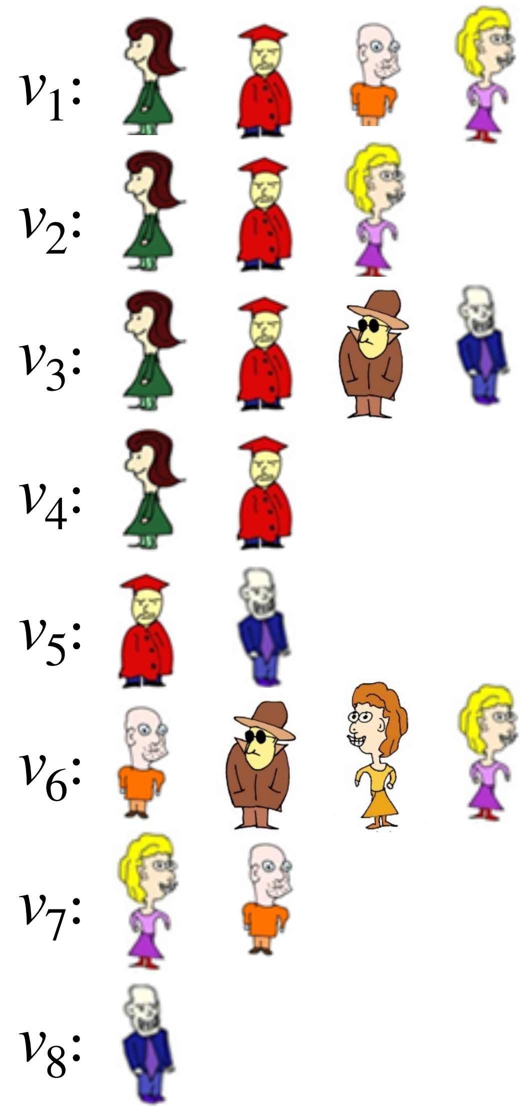
$v_6: 0$

$v_7: 0$



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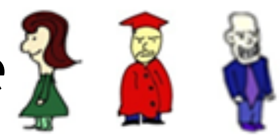
E.g., consider a committee 

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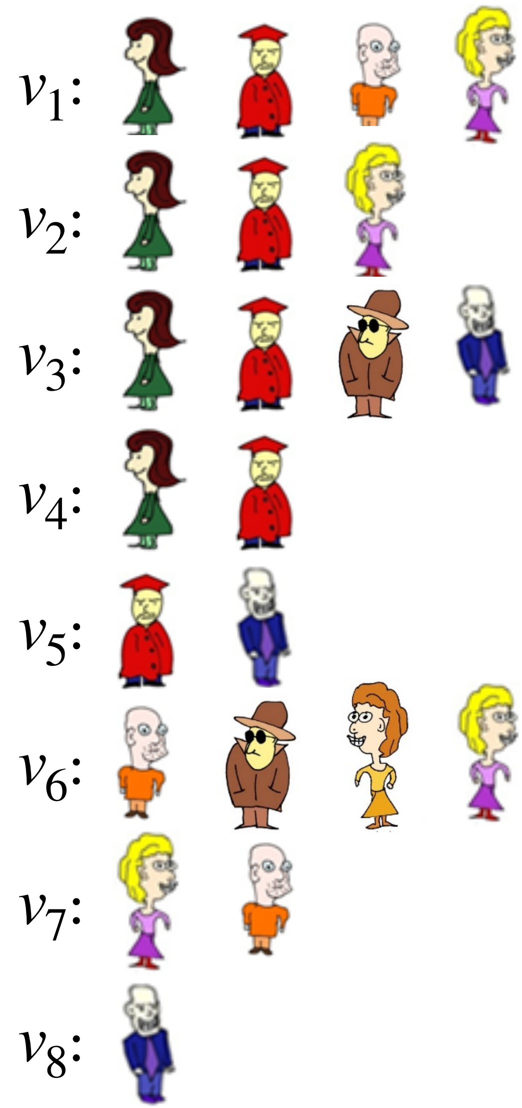
$v_6: 0$

$v_7: 0$

$v_8: 1$



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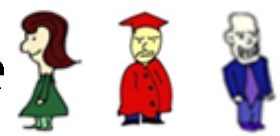
E.g., consider a committee 

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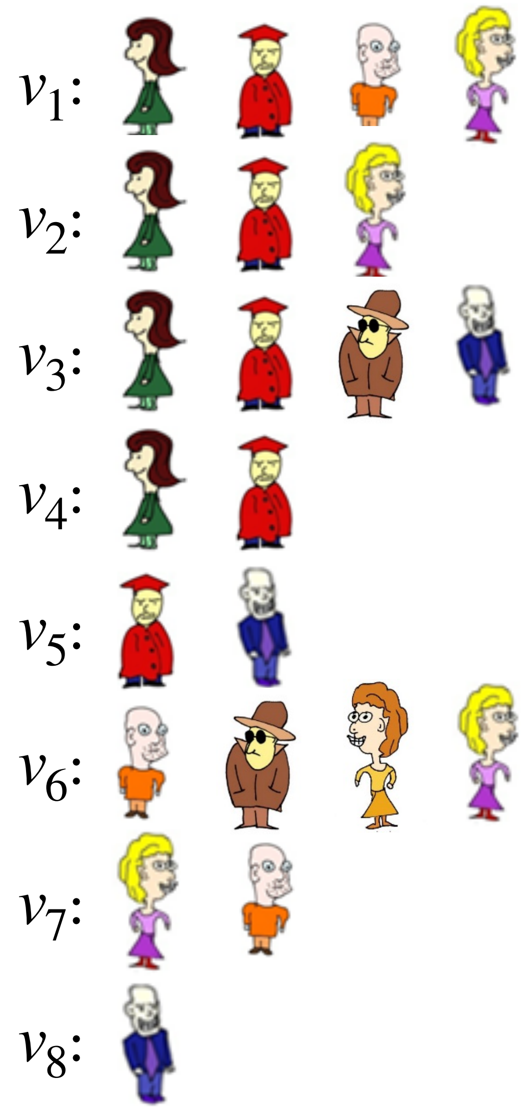
Sum of points = $8+\frac{5}{6}$



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E.g., consider a committee 

Poi

v_1 :

Committee with the highest score wins the election.

$v_3 : 1 + \frac{1}{2} + \frac{1}{3}$ $\quad$ $v_4 : 1 + \frac{1}{2}$

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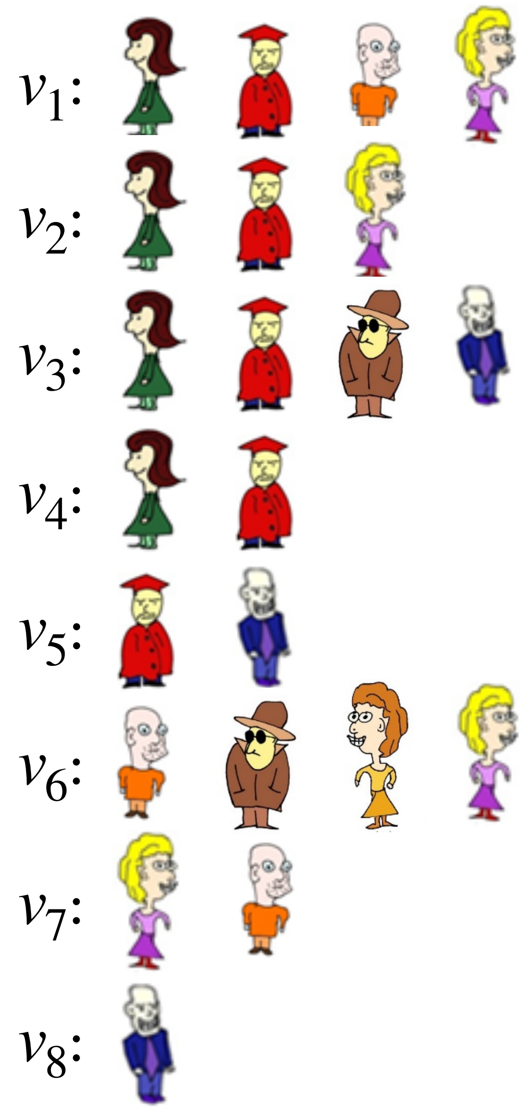

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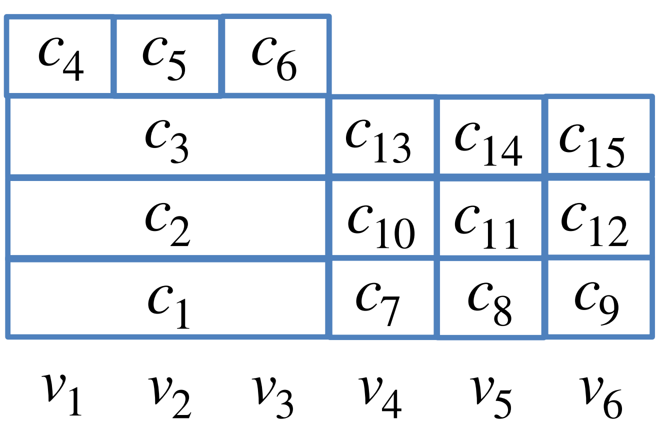

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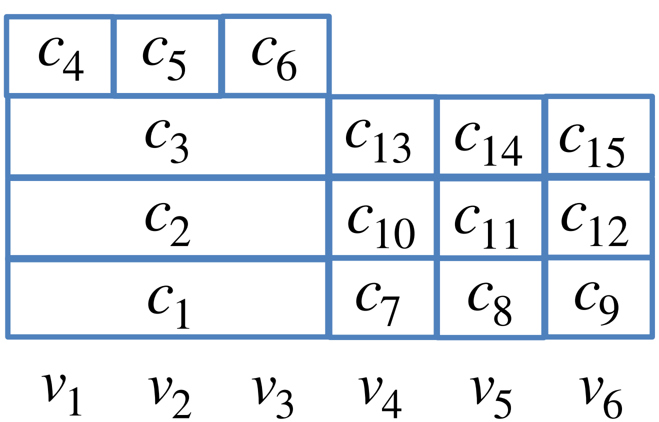
$$k = 12$$



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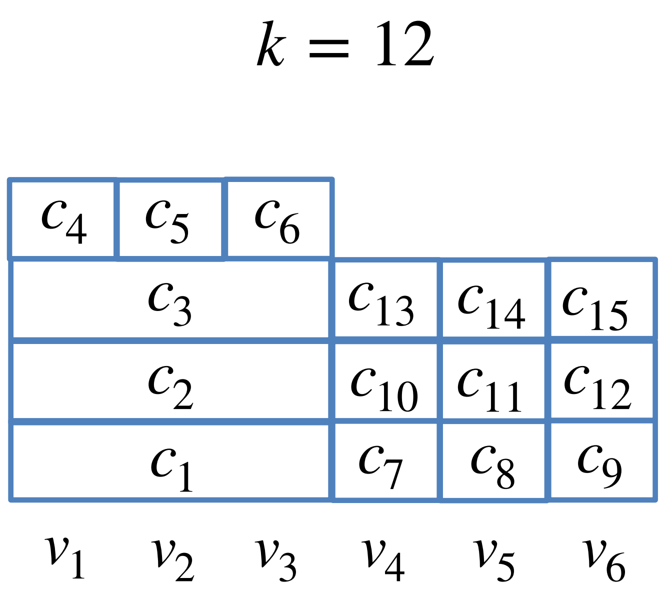

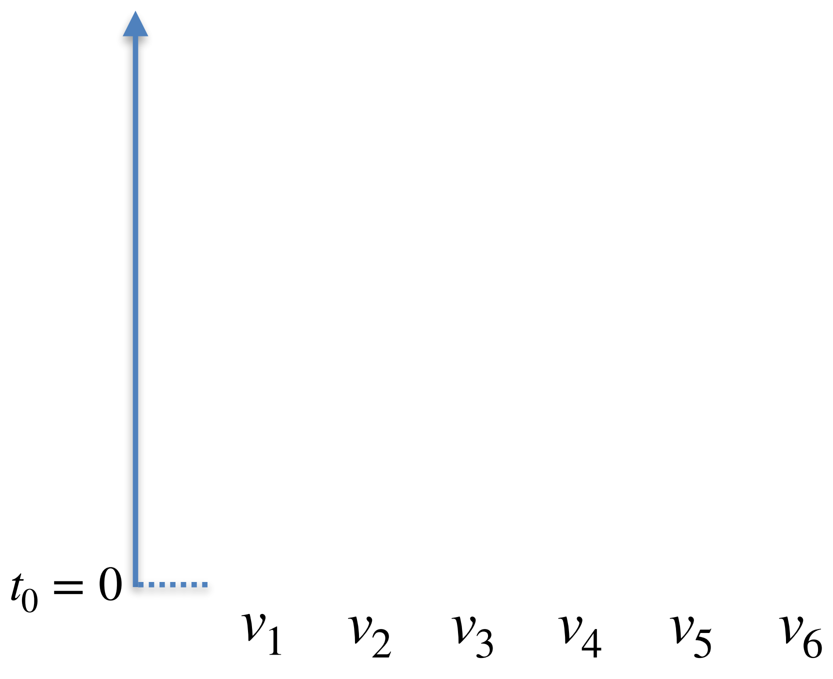

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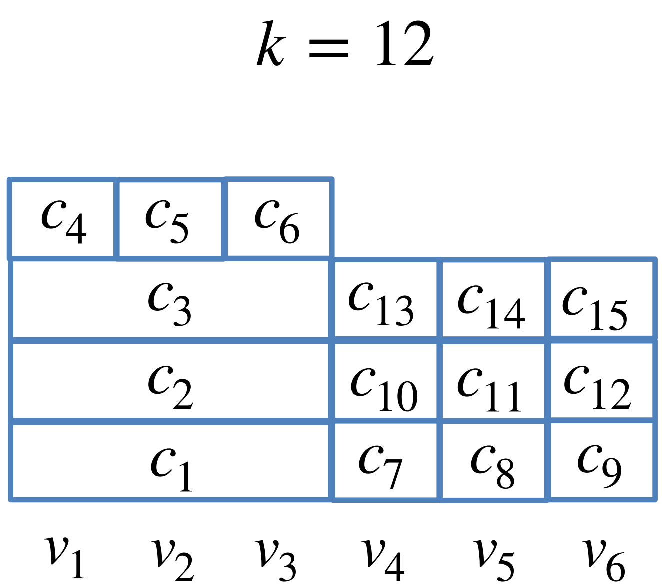

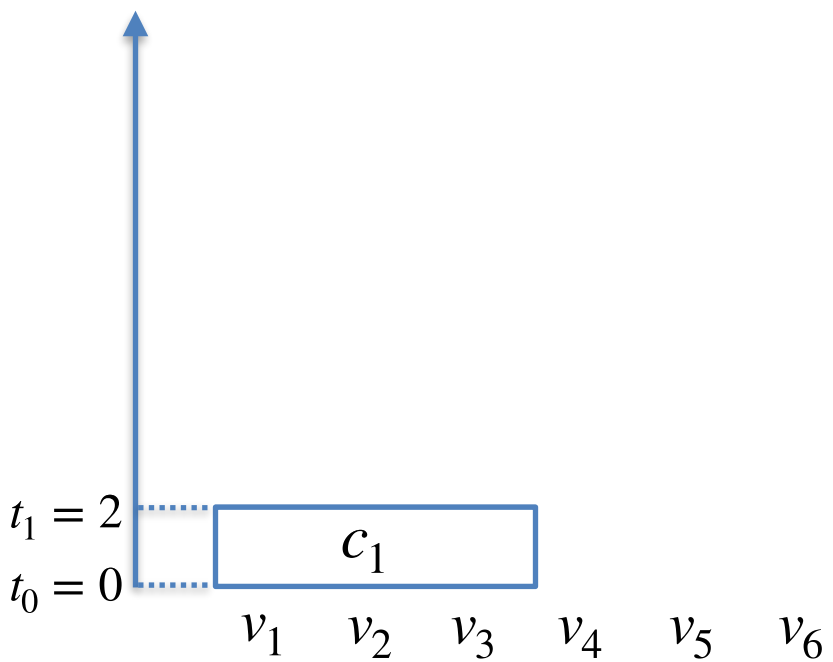

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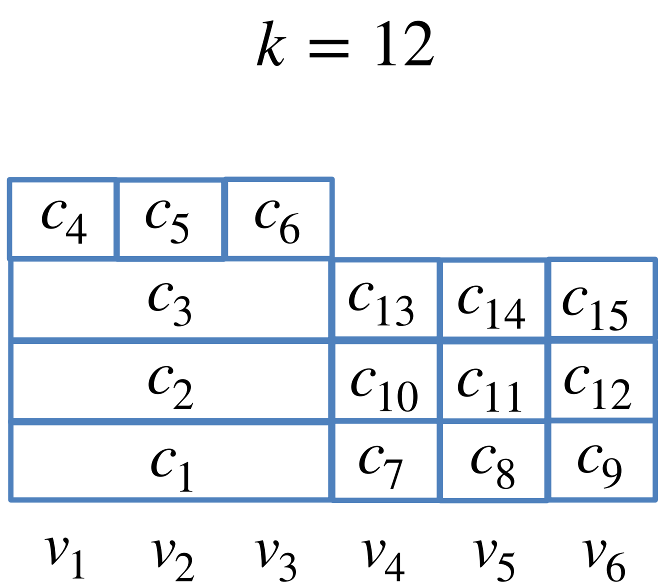

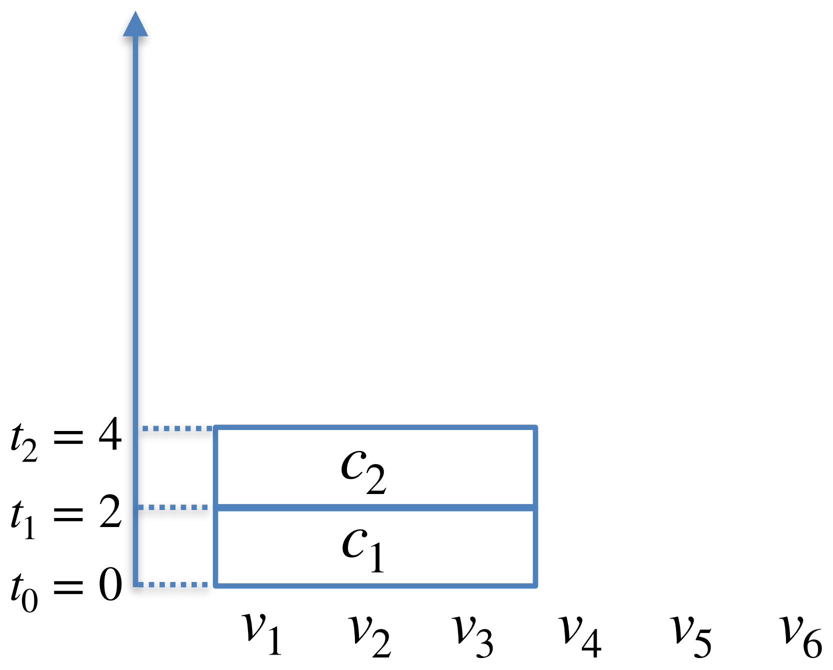

Phragmén's Rule

- Voters earn money with the constant speed ($1 per time unit).
- In the first moment when there is a group of voters S who all have n dollars in total and who all approve a not-yet selected candidate c, do:
  1. Add c to the committee.
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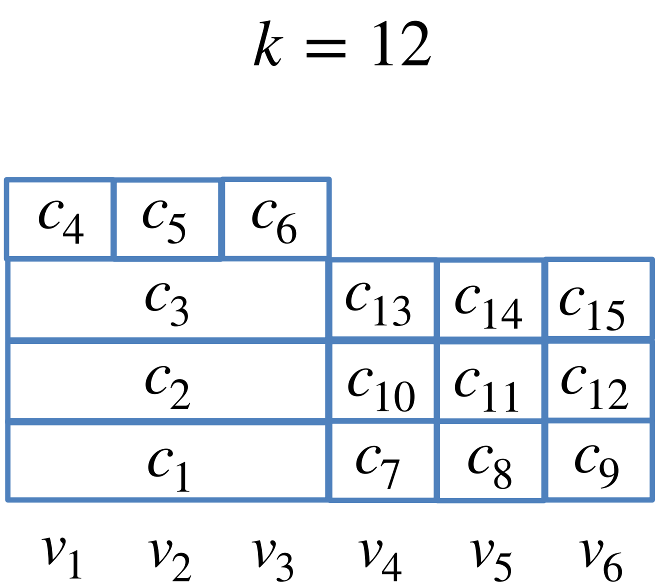

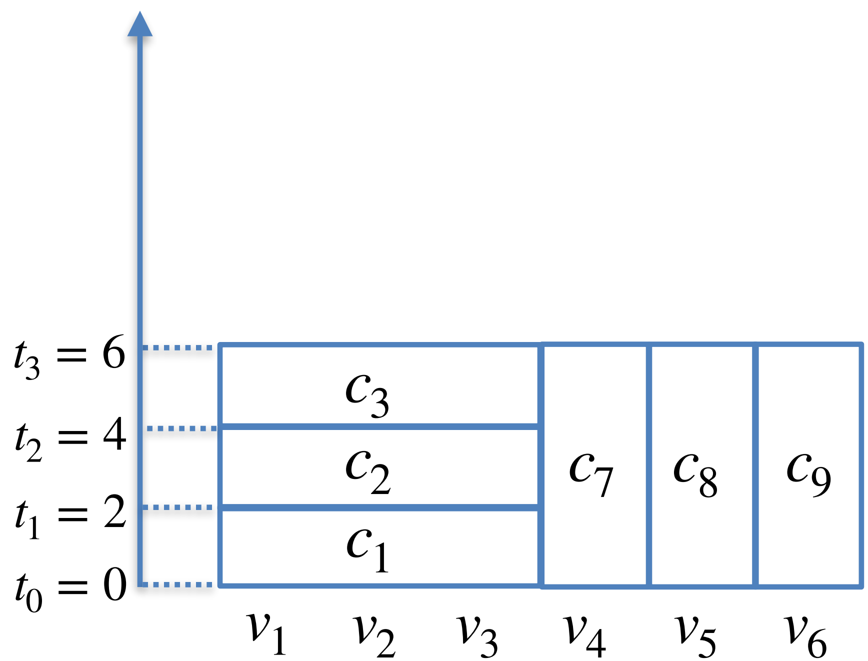

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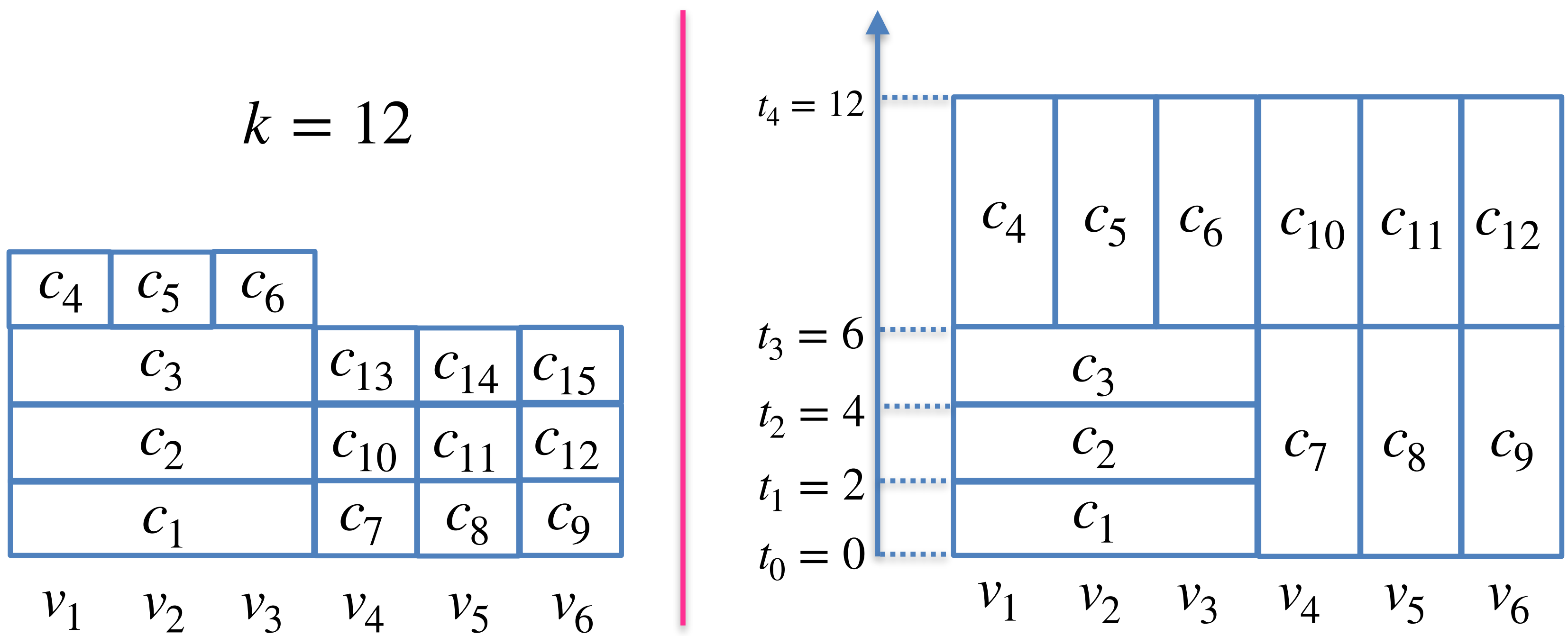

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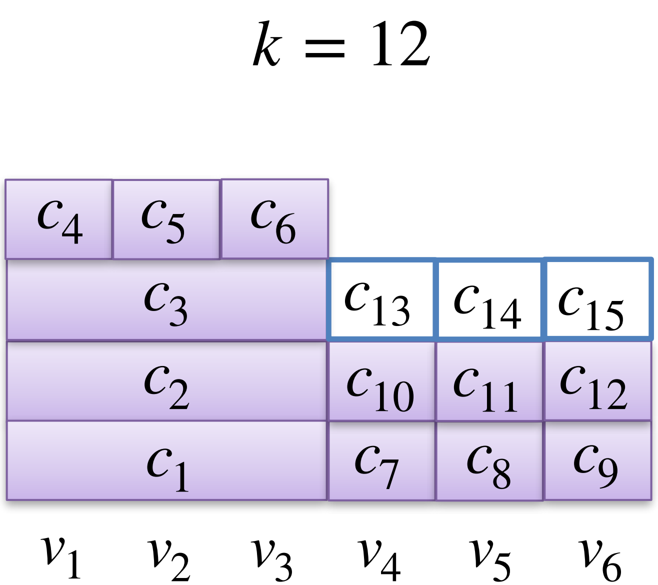

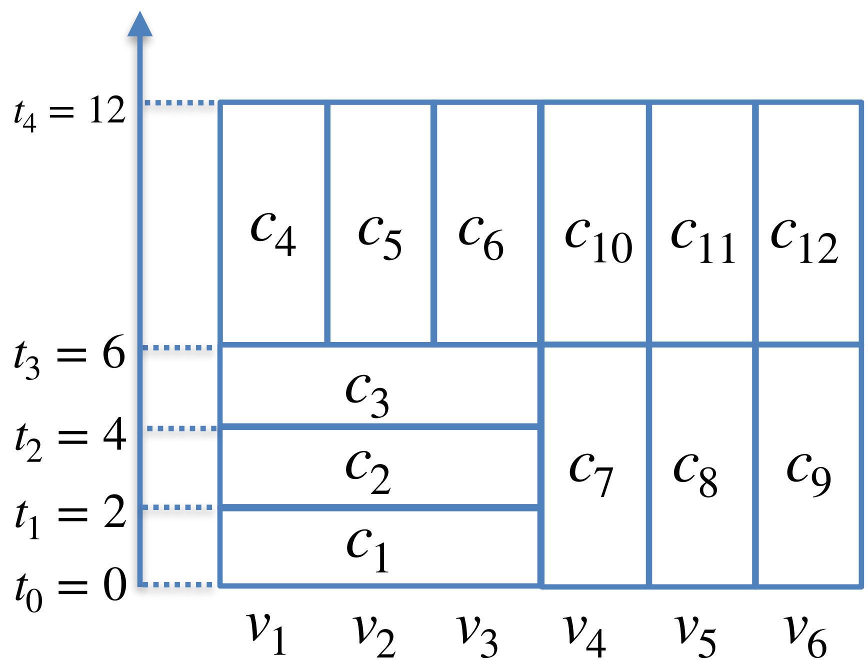

PAV versus Phragmén's Rule

Which of the two rules is better?

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- Both Thiele and Phragmén argued that their rules are proportional by how they behave on party-list profiles.

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PAV versus Phragmén’s Rule

Which of the two rules is better?

- Both Thiele and Phragmén argued that their rules are proportional by how they behave on party-list profiles.
- Historically PAV was preferred since it appeared simpler.
- Current research suggest that PAV is better in terms of proportionality.

Two Arguments in Favour of PAV

First Argument: Axioms for Cohesive Groups

How to define proportionality for more complex preferences?

v_1:

v_2:

v_3:

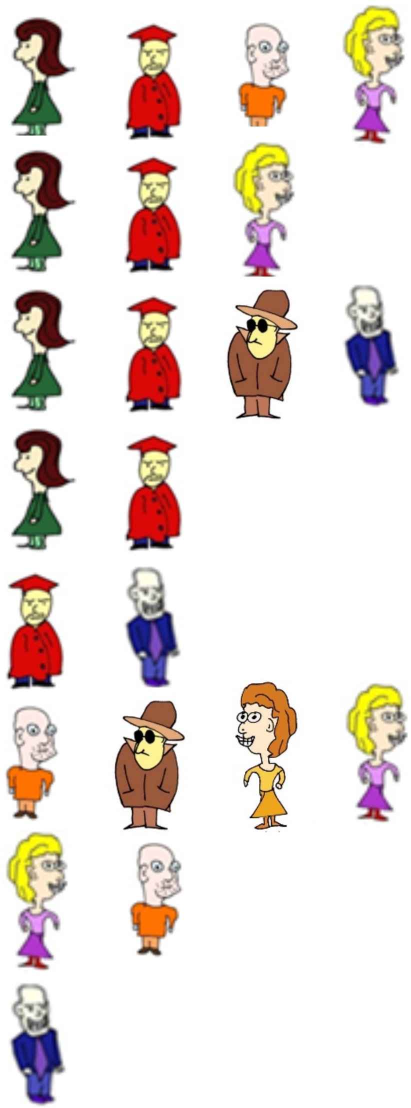
v_4:

v_5:

v_6:

v_7:

v_8:



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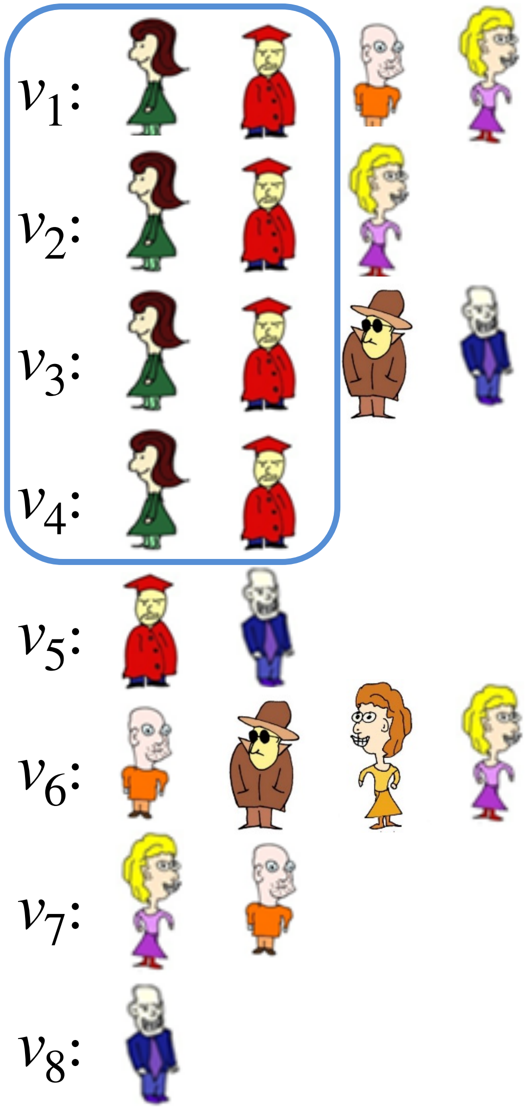

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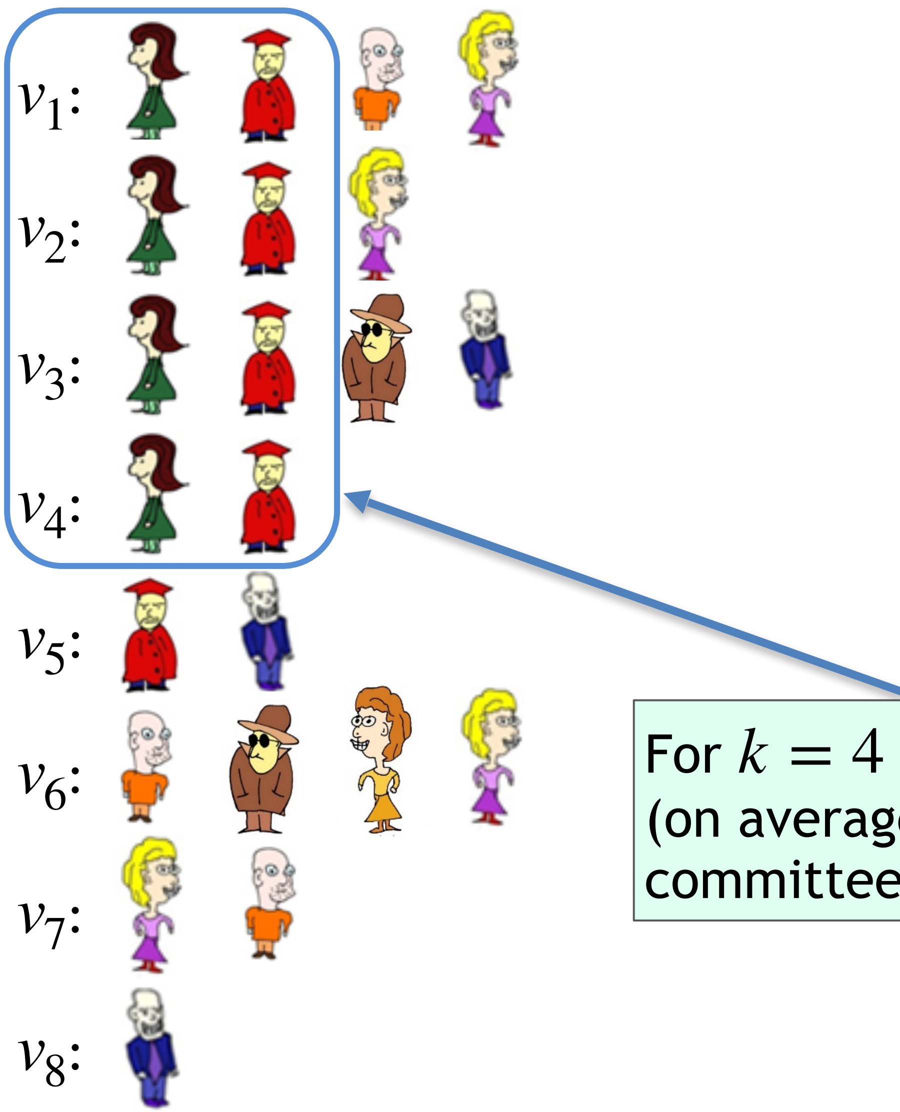

For $k = 4$ these voters should approve (on average) 2 candidates in the selected committee.

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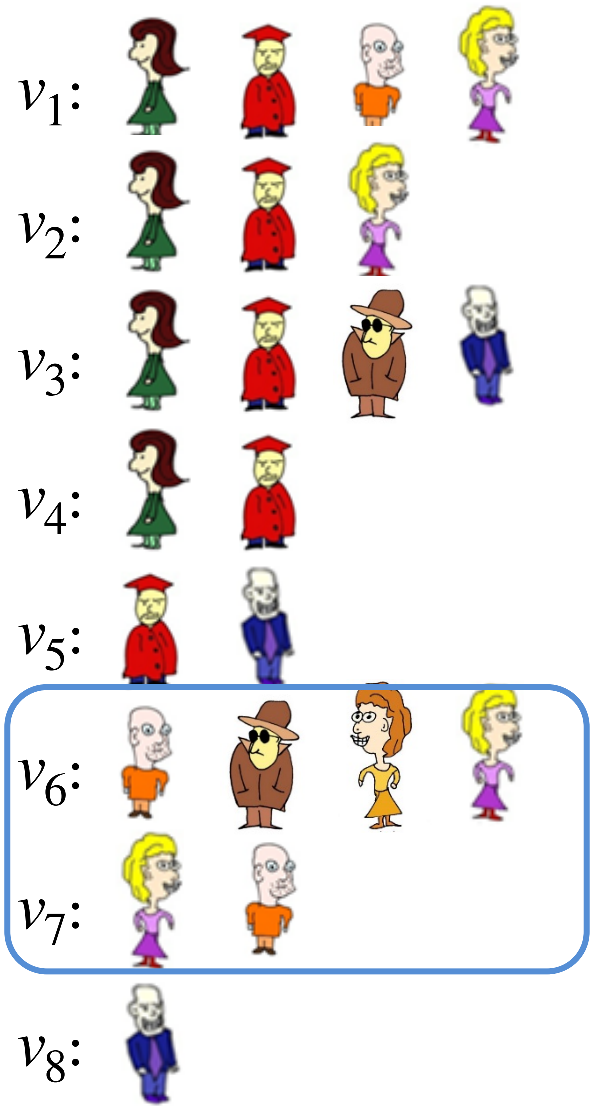

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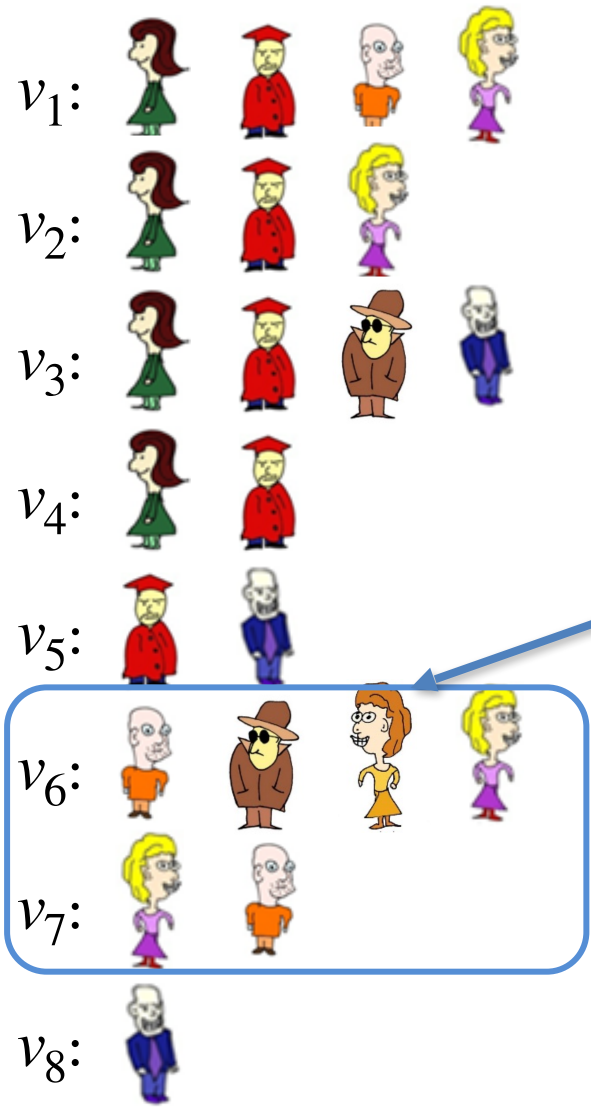
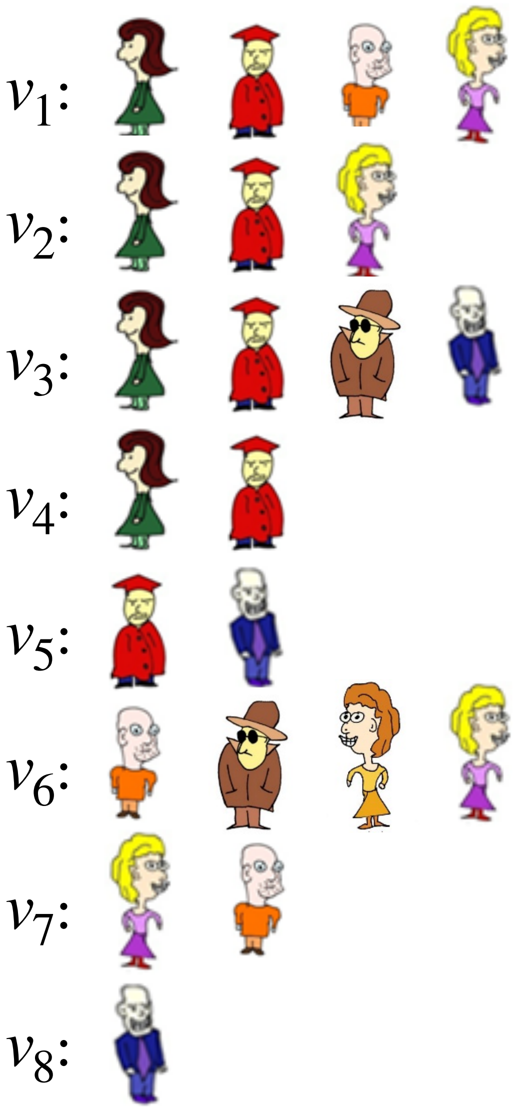


For $k = 4$ these voters should approve (on average) 1 candidate in the selected committee.

How to define proportionality for more complex preferences?



Definition: Each group with at least $\ell n/k$ voters who approve at least ℓ same candidates should have on average at least ℓ representatives in the elected committee.

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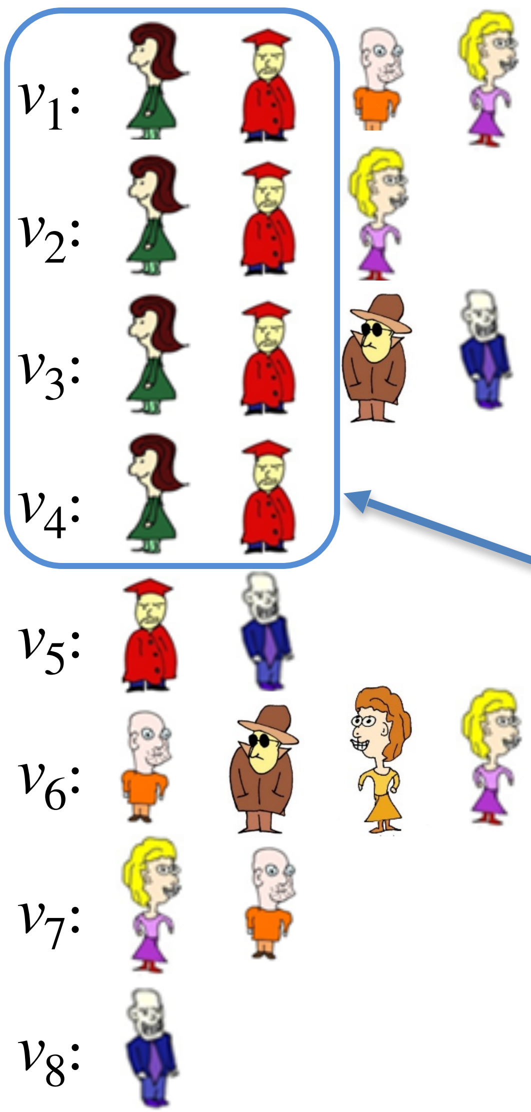

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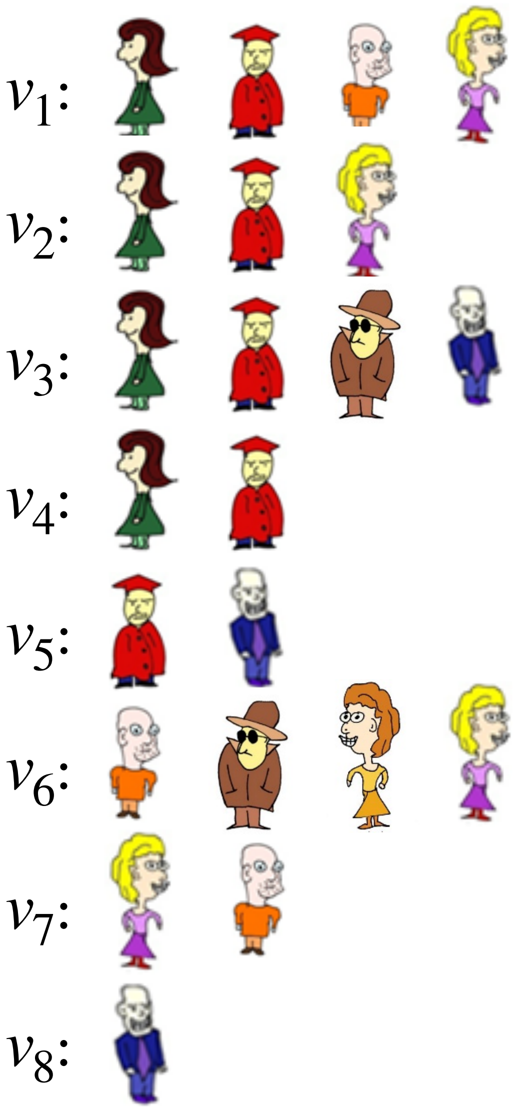

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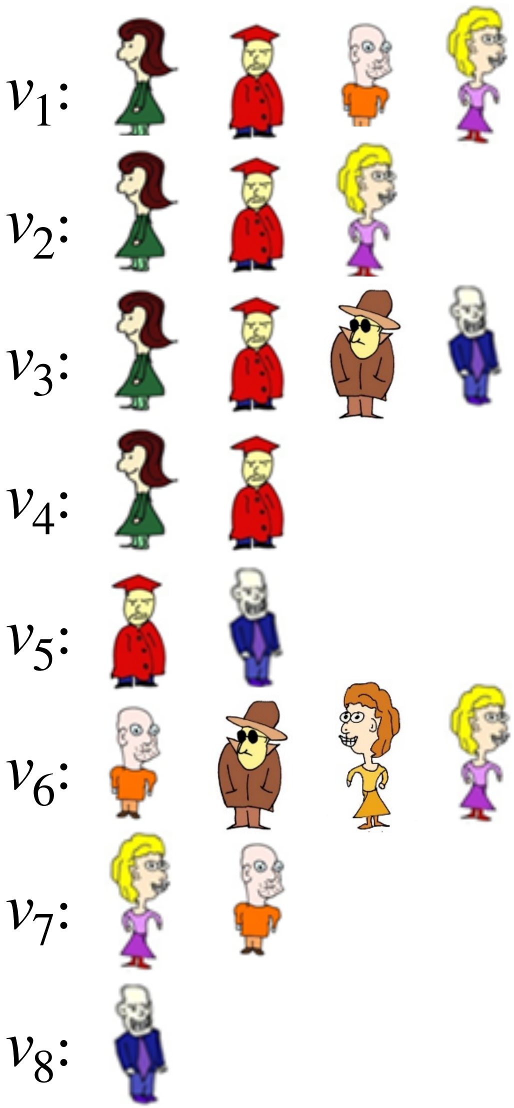

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$v_1: \{a, d\}$	$v_7: \{b, c\}$	
$v_2: \{a\}$	$v_8: \{c\}$	
$v_3: \{a\}$	$v_9: \{c\}$	$n = 12$
$v_4: \{a, b\}$	$v_{10}: \{c, d\}$	$k = 3$
$v_5: \{b\}$	$v_{11}: \{d\}$	
$v_6: \{b\}$	$v_{12}: \{d\}$	

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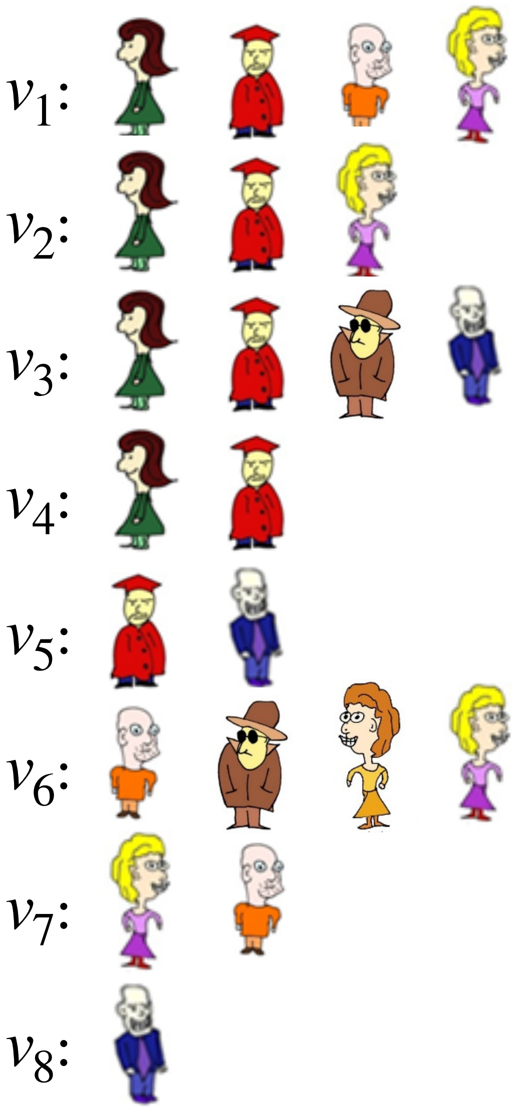

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But PAV satisfies a slightly weaker property!

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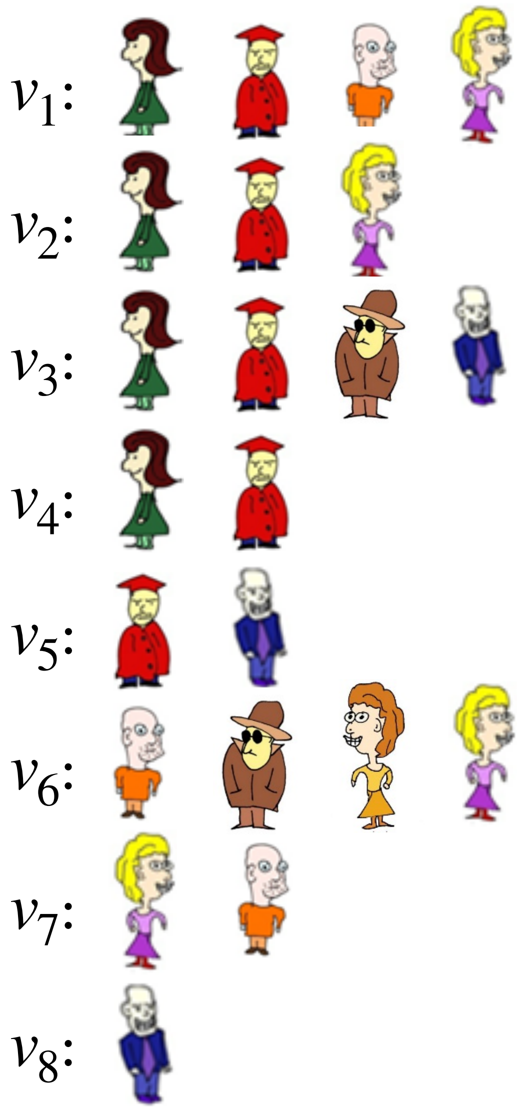

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But PAV satisfies a slightly weaker property!

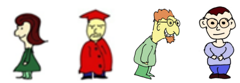
Phragmén’s Rule would satisfy it only if we replaced $\ell - 1$ with $(\ell - 1)/2$.

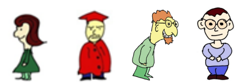
Two Arguments in Favour of PAV

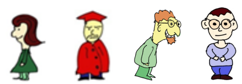
Second Argument: Axiomatic Extensions of Apportionment Methods

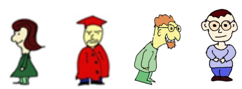
Let's look at this instance

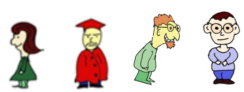
We have 9 voters, 9 candidates, and our goal is to select a committee of size $k = 4$.

v_1: 

v_2: 

v_3: 

v_4: 

v_5: 

v_6: 

v_7: 

v_8: 

v_9: 

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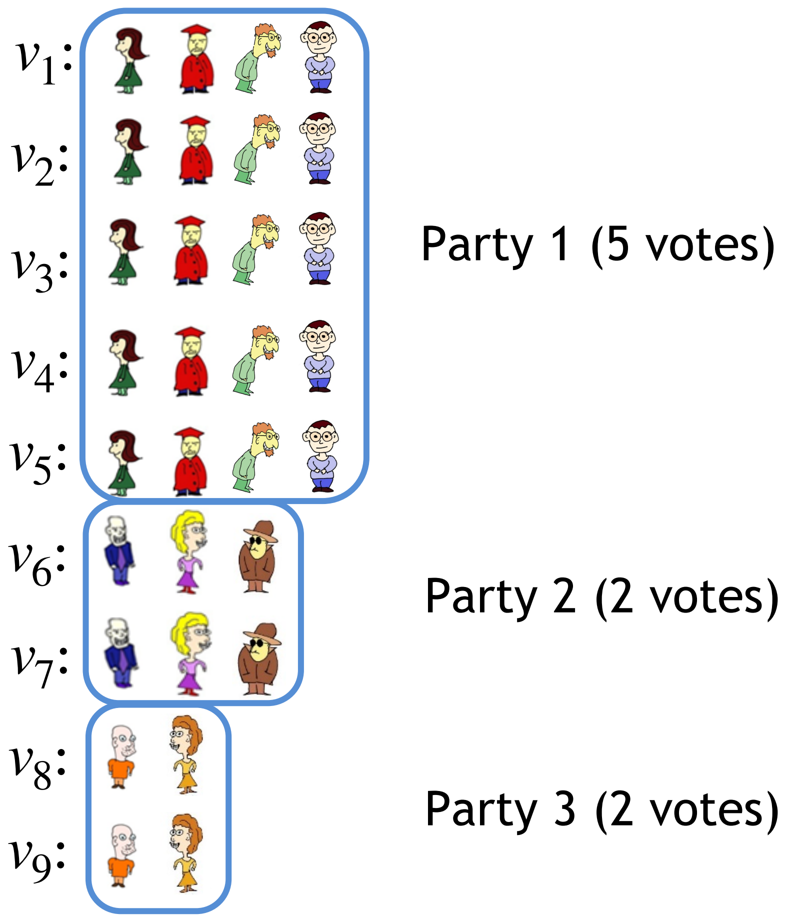

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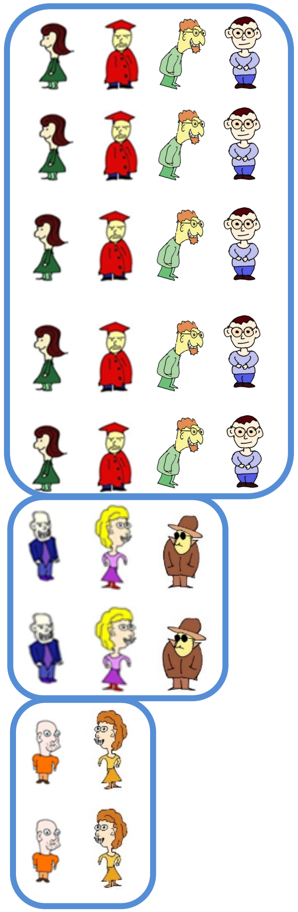


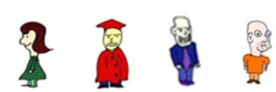
Party 1 (5 votes)

Party 2 (2 votes)

Party 3 (2 votes)

- Party 1 gets 2 seats.
- Party 2 gets 1 seat.
- Party 3 gets 1 seat.

For example



Some basic axiomatic properties: **Symmetry**

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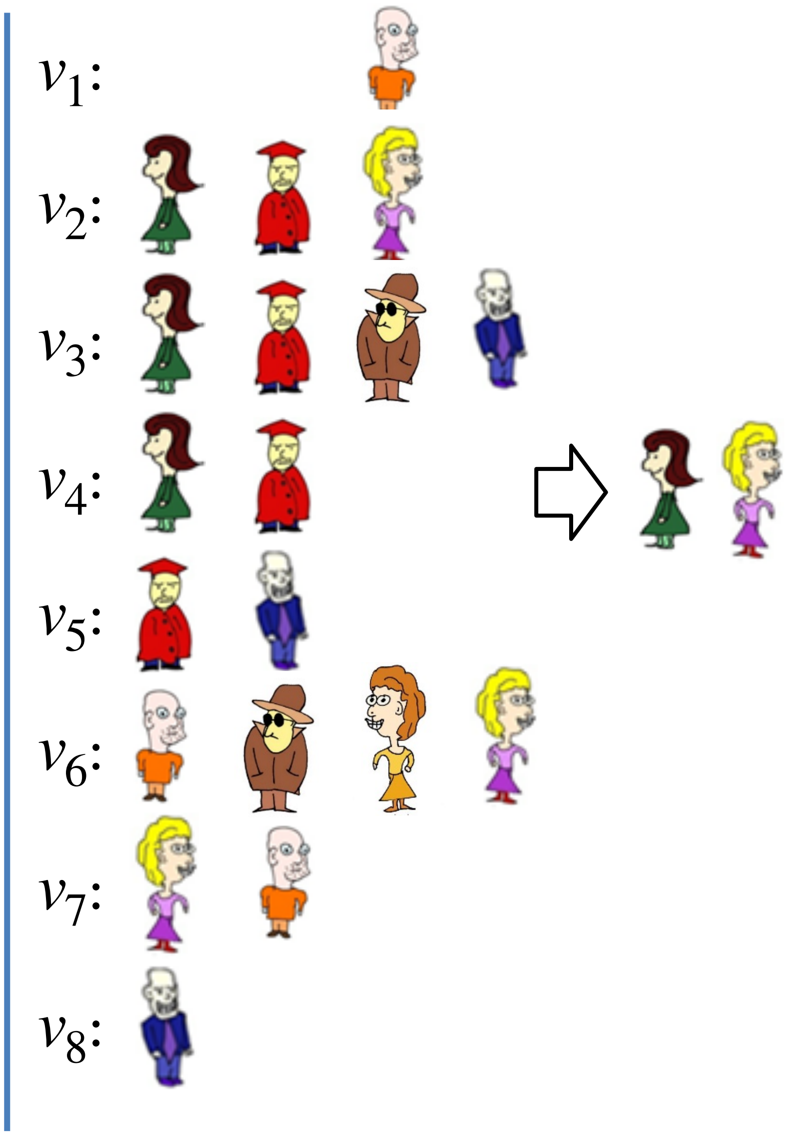

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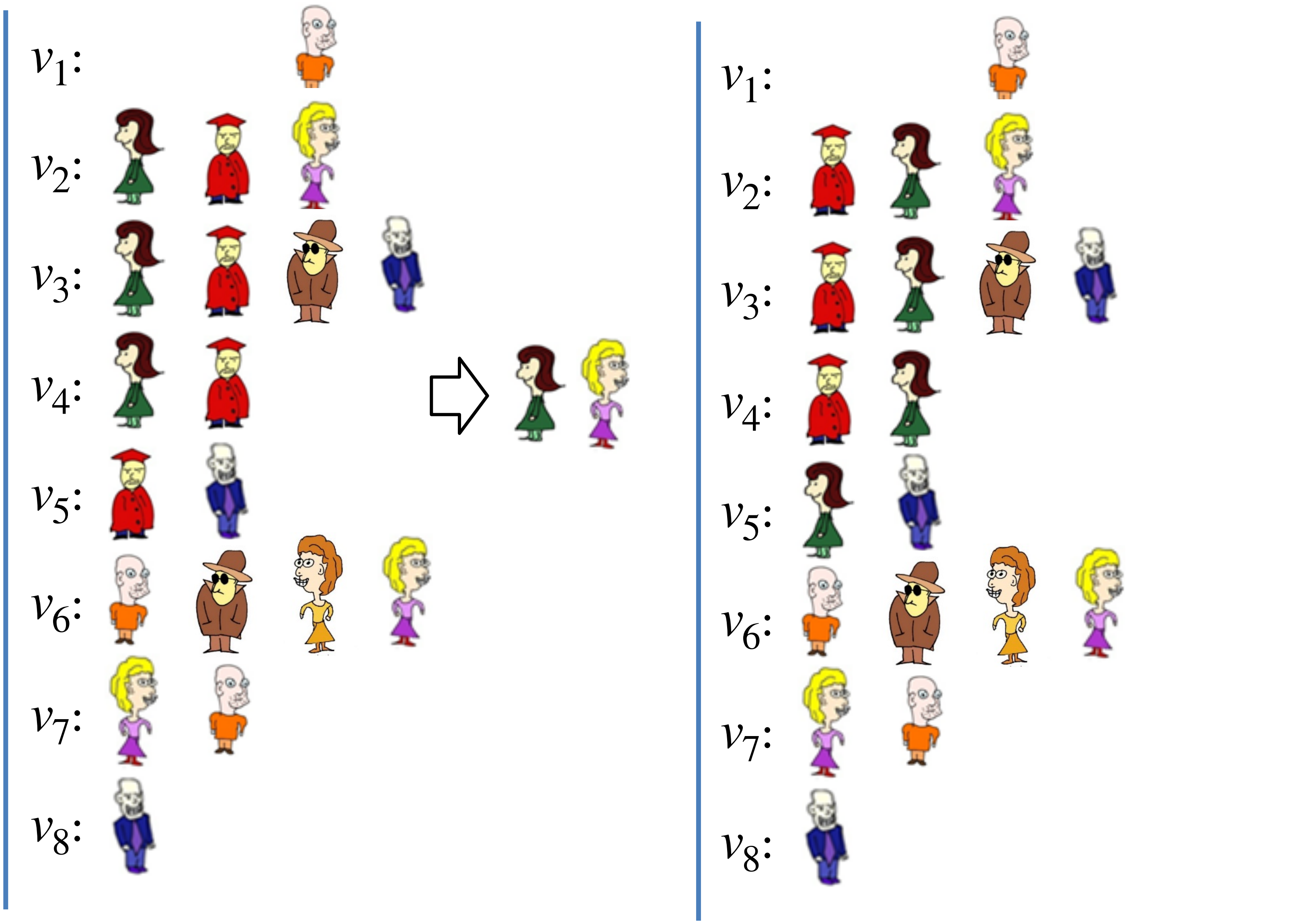

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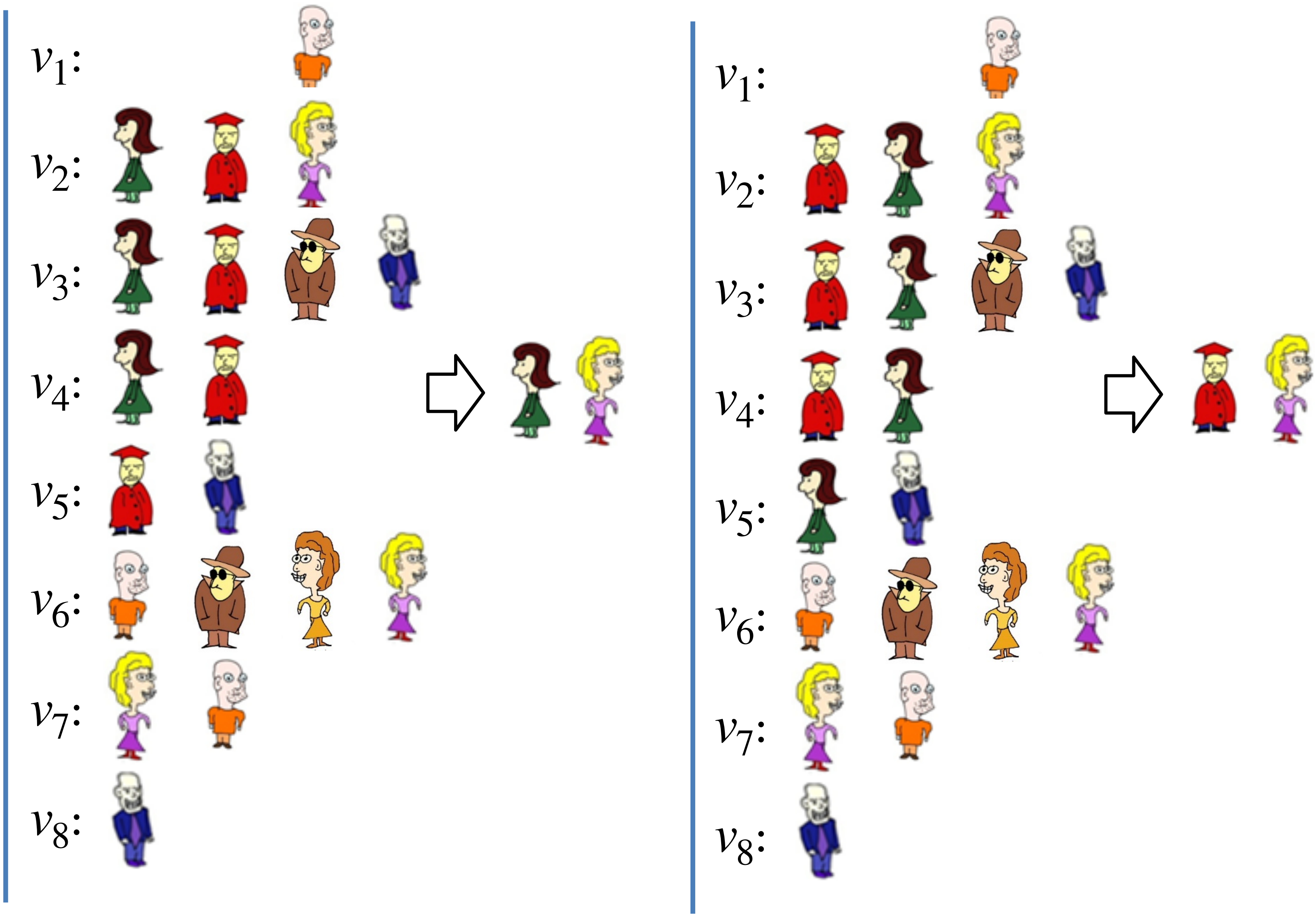

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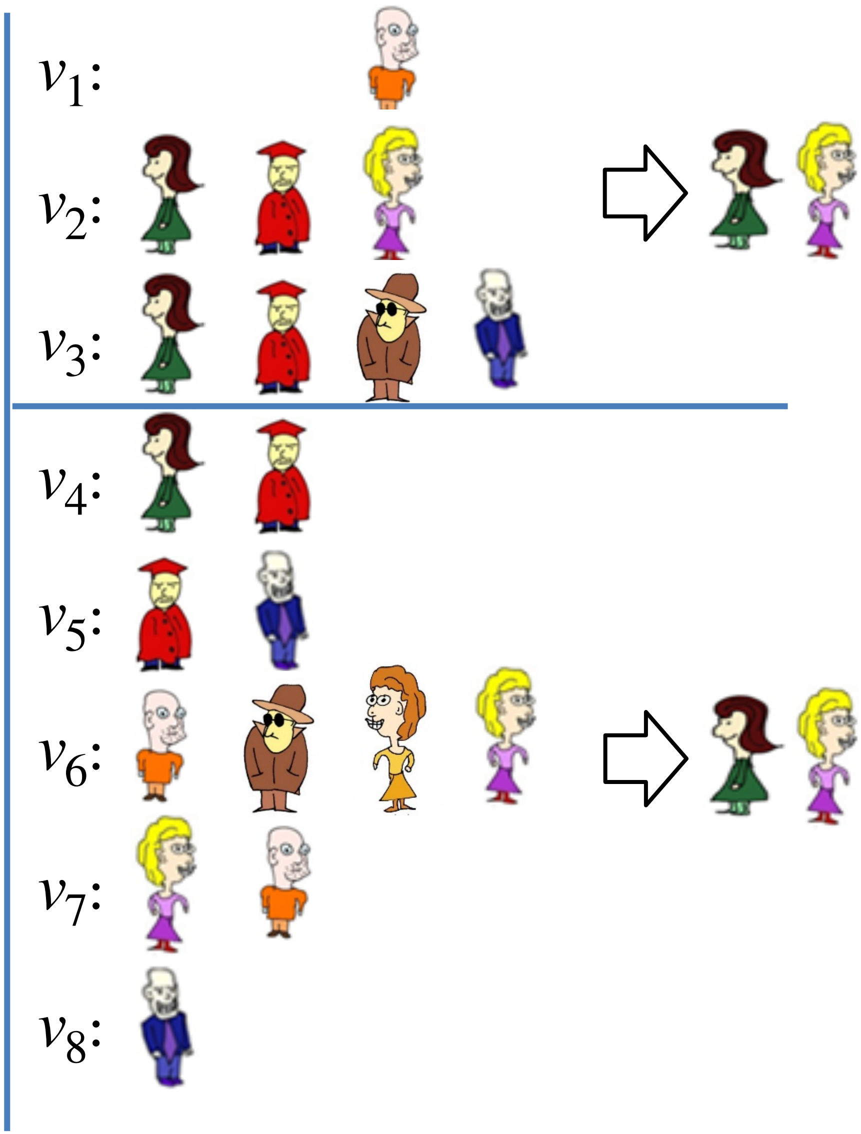

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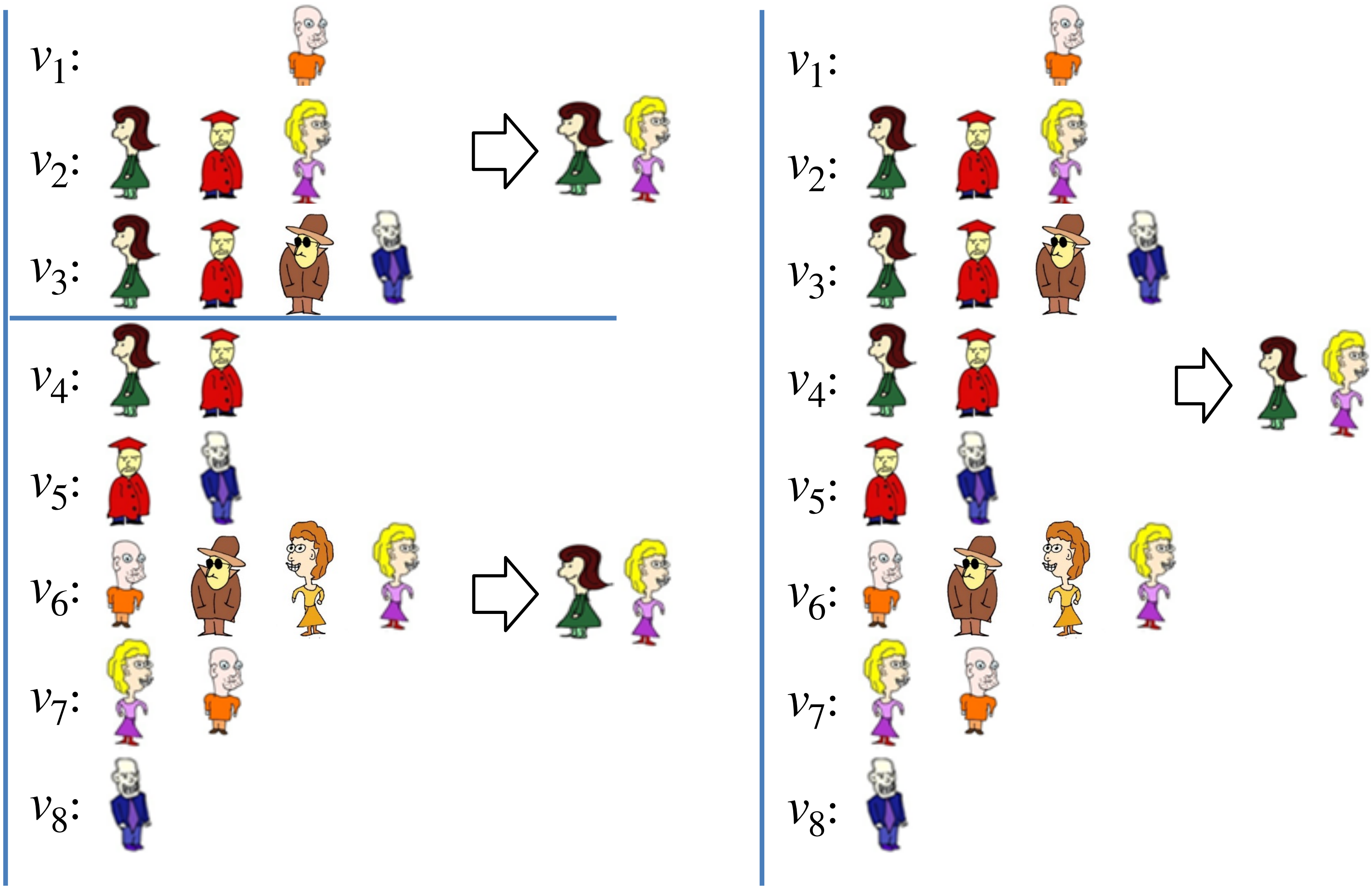

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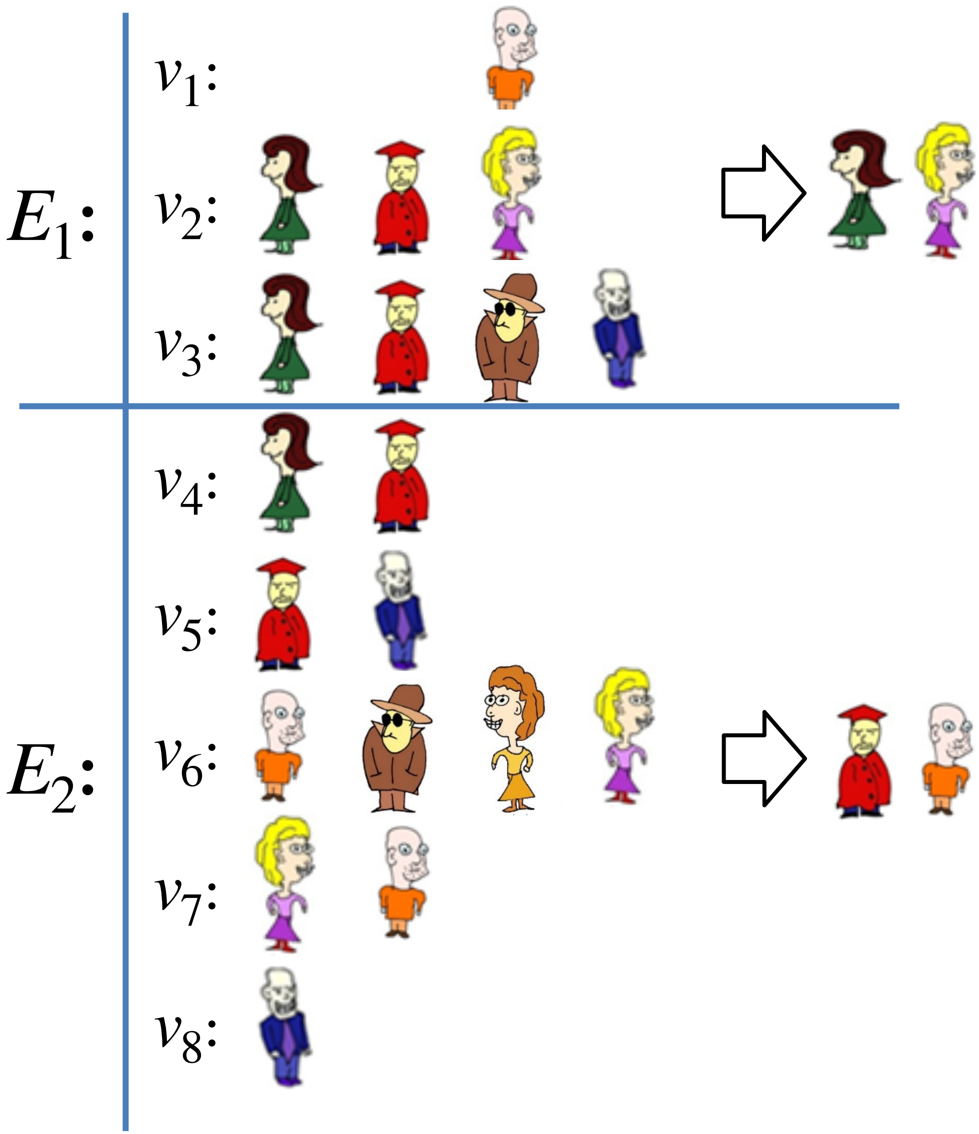

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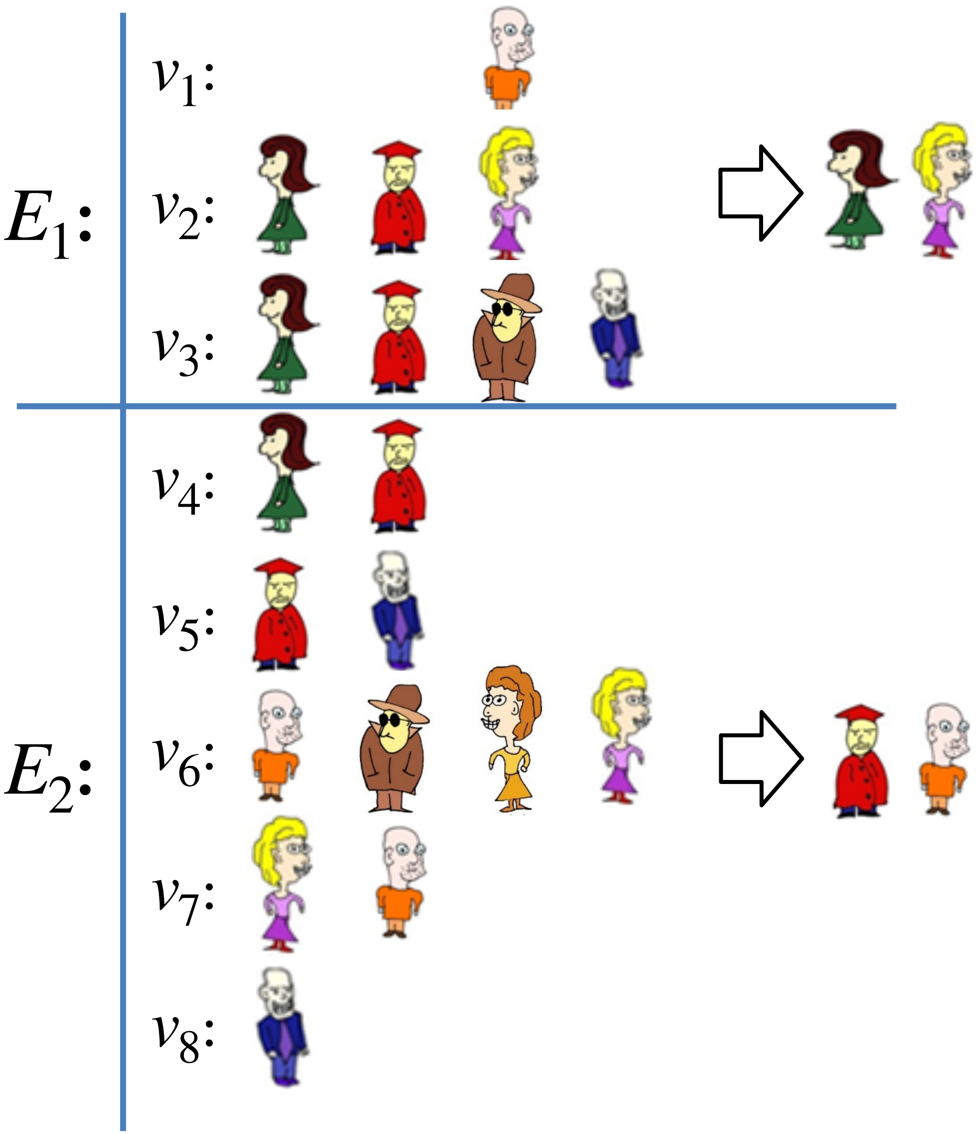
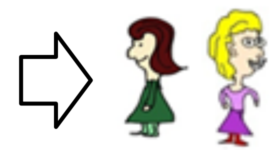


Then, there exists (possibly very large) value z such that:

$$z \cdot E_1 + E_2$$



Axiomatic Characterisations

Theorem: Proportional Approval Voting is the only ABC ranking rule that satisfies symmetry, consistency, continuity and D'Hondt proportionality.

[LS17] M. Lackner, P. Skowron, Consistent Approval-Based Multi-Winner Rules, Arxiv 2017.

Axiomatic Characterisations

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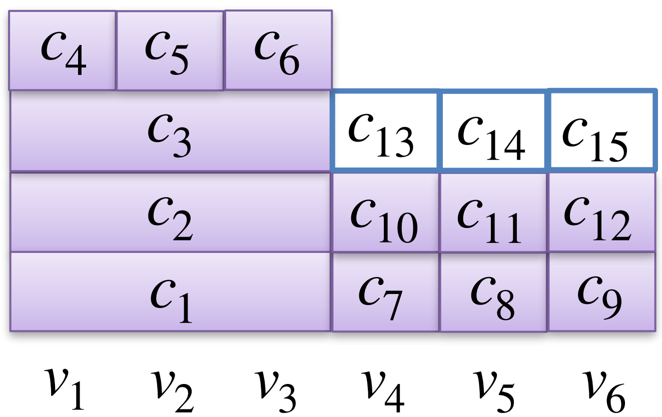
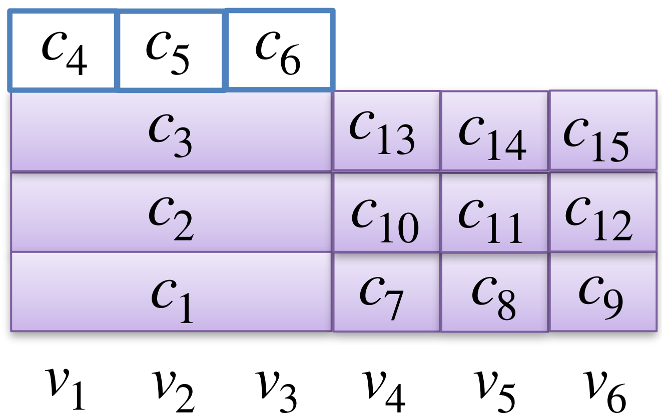
$$k = 12$$



Phragmén's Rule



Thiele's Rule (PAV)

PAV versus Phragmén's Rule

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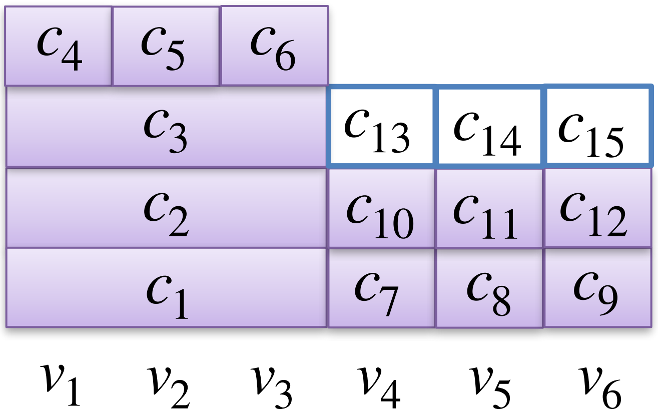

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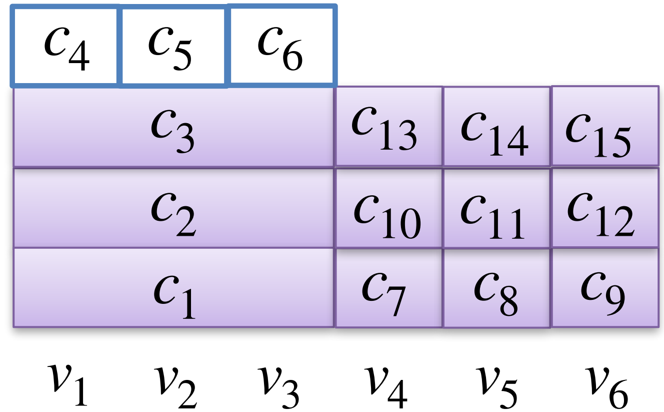

Thiele's Rule (PAV)

Proportionality with respect to power

Proportionality with respect to welfare

PAV versus Phragmén's Rule

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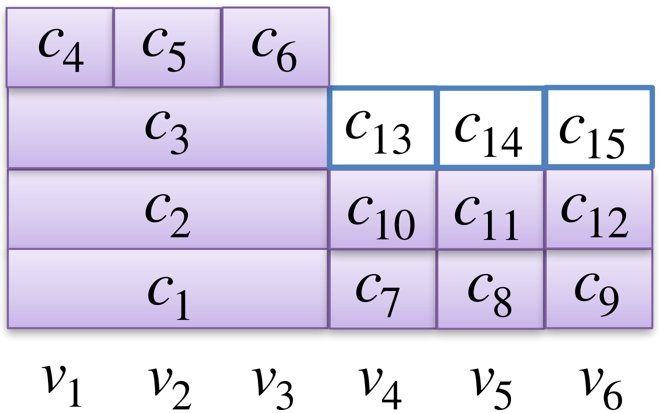

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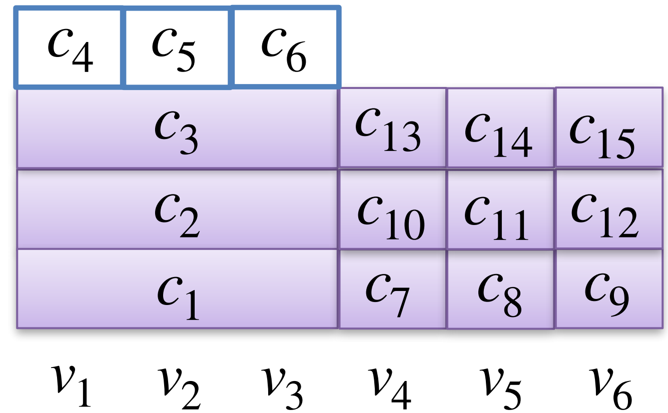

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Proportionality with respect to power

- priceability,
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PAV versus Phragmén’s Rule

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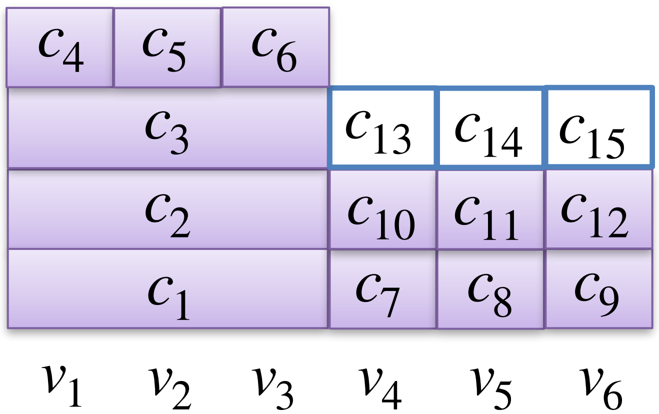

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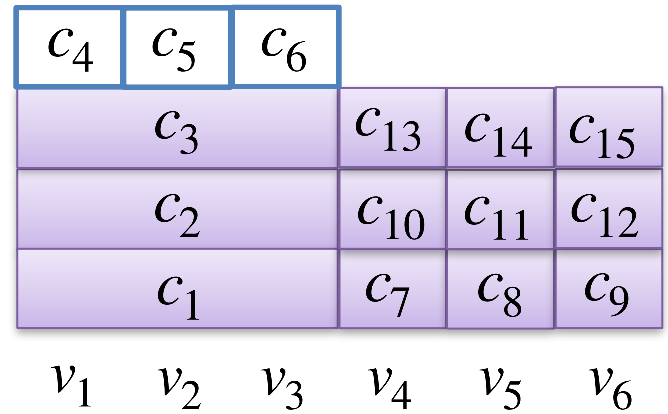

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Two New Notions of Proportionality

Fair distribution of power

(failed by PAV)

Laminar Proportionality: Examples

It describes how the rule should behave on certain well-behaved profiles

Laminar Proportionality: Examples

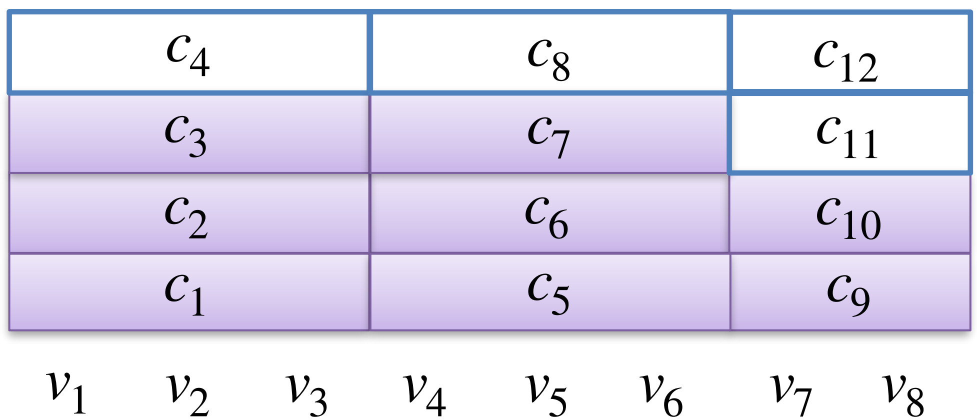
$$k = 8$$

c_4	c_8	c_{12}
c_3	c_7	c_{11}
c_2	c_6	c_{10}
c_1	c_5	c_9
v_1 v_2 v_3	v_4 v_5 v_6	v_7 v_8

Party list profiles

Laminar Proportionality: Examples

$$k = 8$$



Party list profiles

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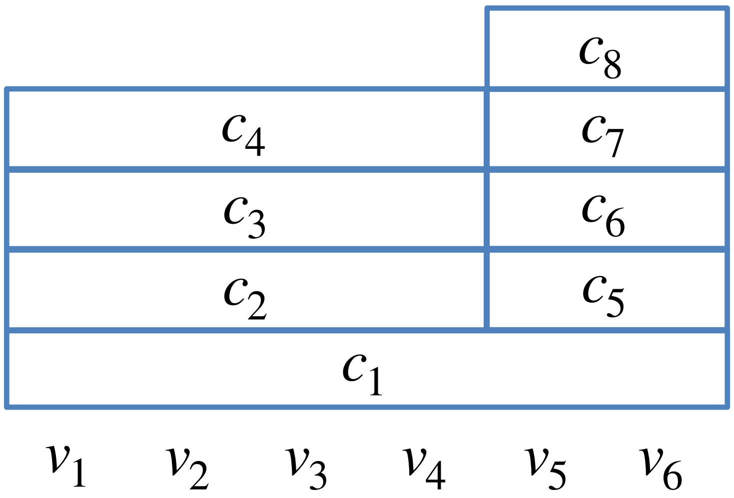

Party lists with a common leader

Laminar Proportionality: Examples

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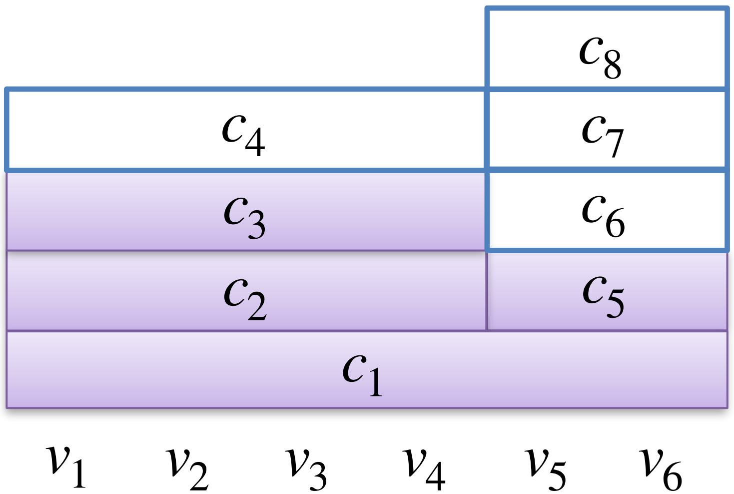

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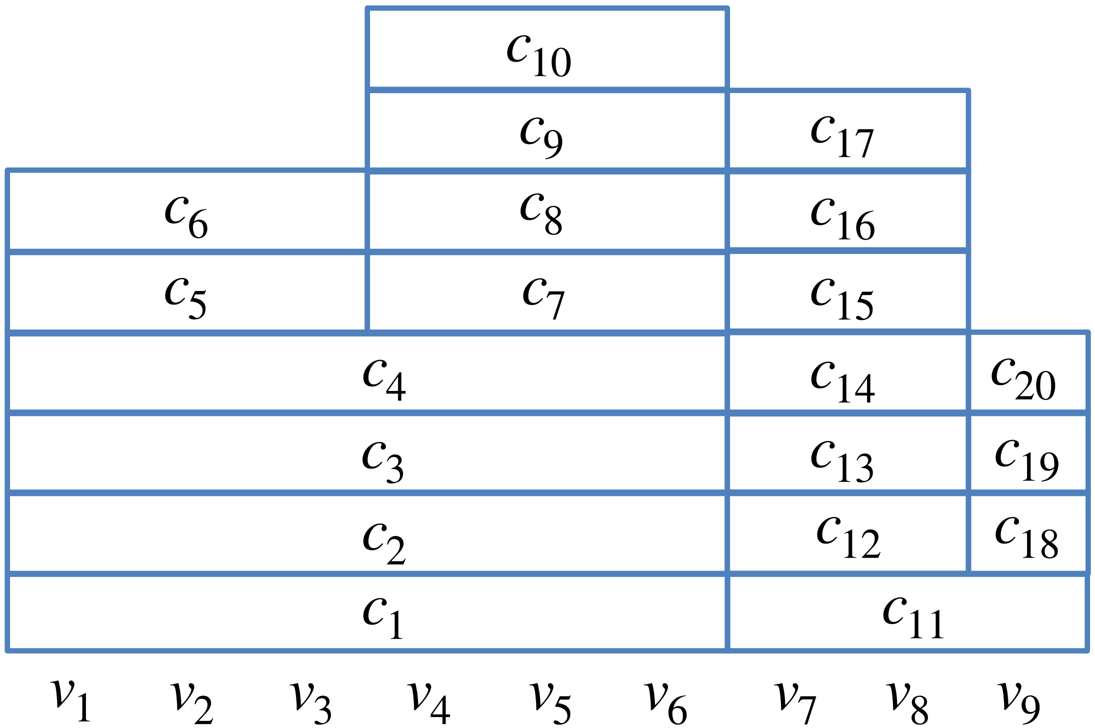

Subdivided parties

Laminar Proportionality: Examples

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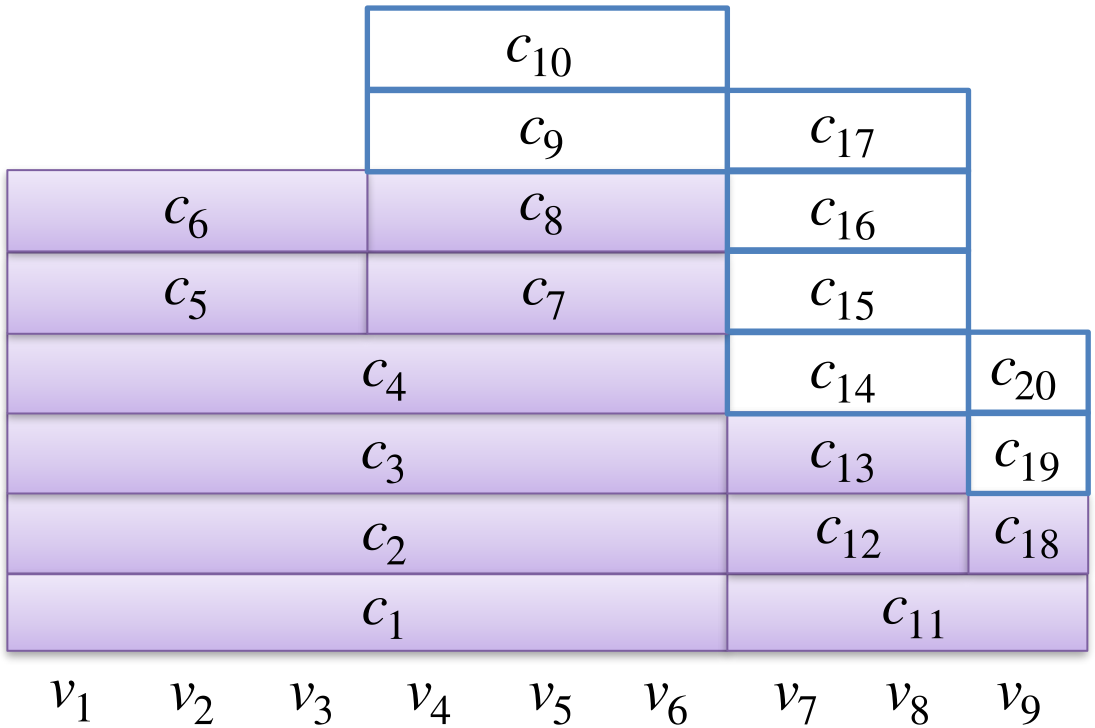

Subdivided parties

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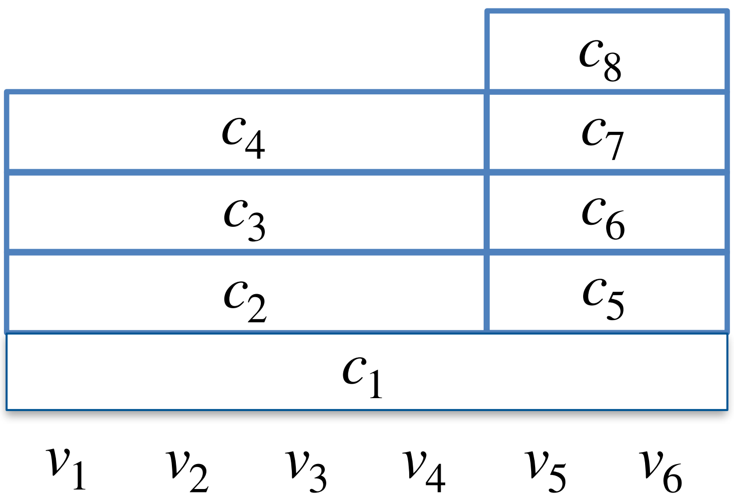

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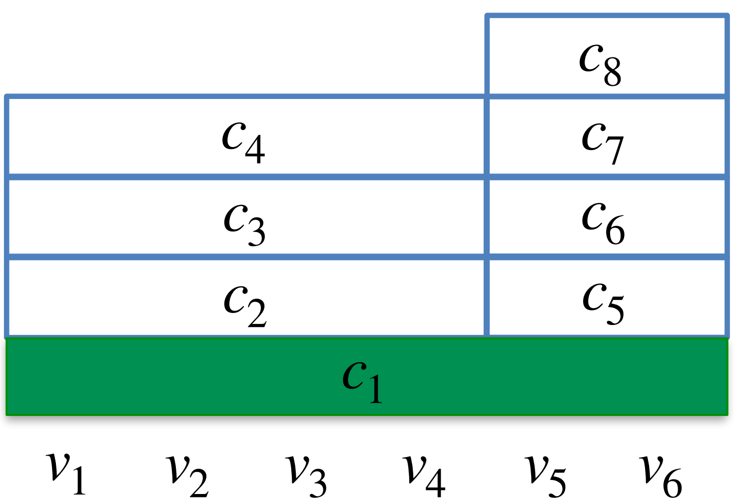

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3. There are two disjoint laminar instances (P_1, k_1) and (P_2, k_2) with $\frac{|P_1|}{k_1} = \frac{|P_2|}{k_2}$ such that $P = P_1 + P_2$ and $k = k_1 + k_2$

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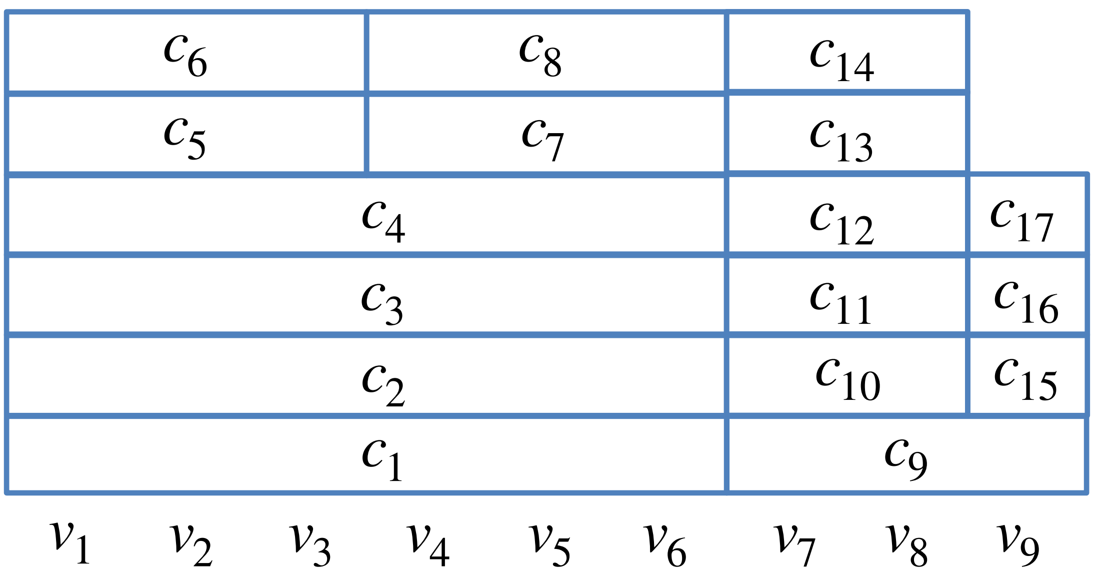

Laminar Proportionality: Definition

We say that a profile (P, k) is **laminar** if:

1. P is unanimous, or
2. There exists a **unanimously approved** candidate c, and $(P \setminus \{c\}, k-1)$ is laminar, or
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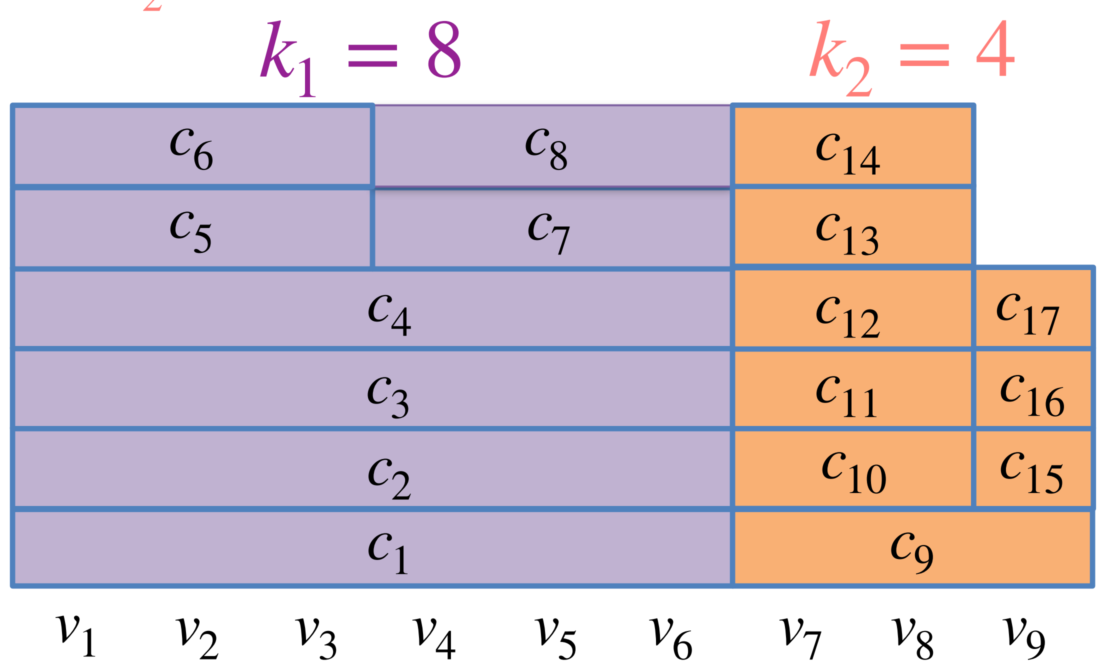

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We say that a rule is laminar proportional if it behaves well on laminar profiles.

Priceability

Priceability

A price system is a pair $\mathrm{ps} = (p, \{p_i\}_{i \in [n]})$, where $p > 0$ is a price, and for each voter $i \in [n]$, there is a payment function $p_i \colon C \to [0,1]$ such that:

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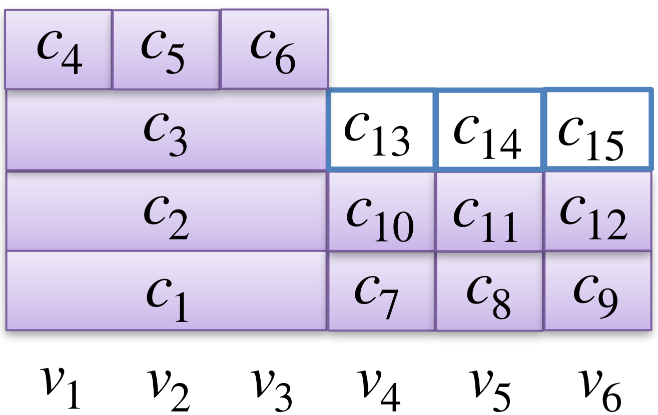
3. There exists no unelected candidate whose supporters, in total, have a remaining unspent budget of more than p

Priceability: Example

$$k = 12$$



Phragmén's Rule

The price is $p = 0.5$.

1. v_1 pays $\frac{1}{6}$ for c_1, c_2 and c_3 and $\frac{1}{2}$ for c_4.

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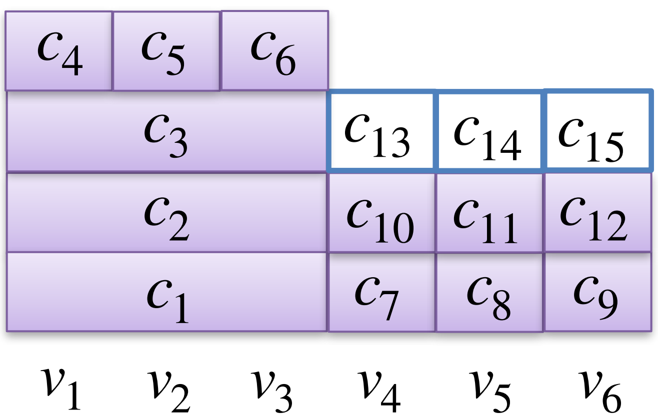

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3. v_3 pays $\frac{1}{6}$ for c_1, c_2 and c_3 and $\frac{1}{2}$ for c_6.

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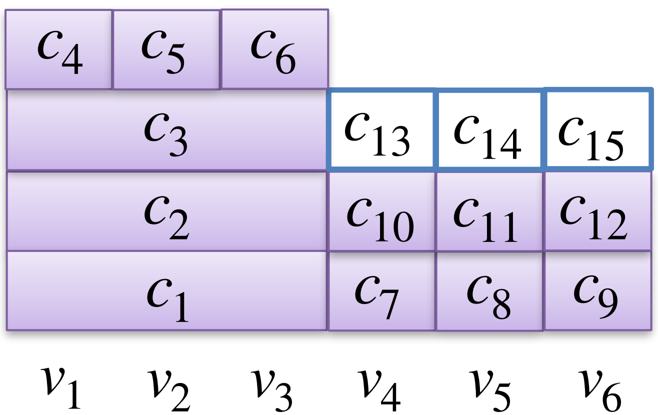

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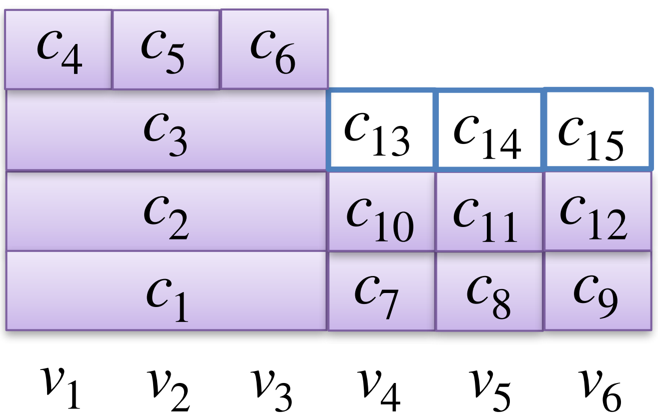

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5. v_5 pays $\frac{1}{2}$ for c_8 and c_{11}.
6. v_6 pays $\frac{1}{2}$ for c_9 and c_{12}.

Core

Core: Definition

We say that a committee W is in the core if there exists no group of voters S and a subset of candidates T such that:

1. $\frac{|T|}{k} \leq \frac{|S|}{n}$, and
2. Each voter in S prefers T to W.

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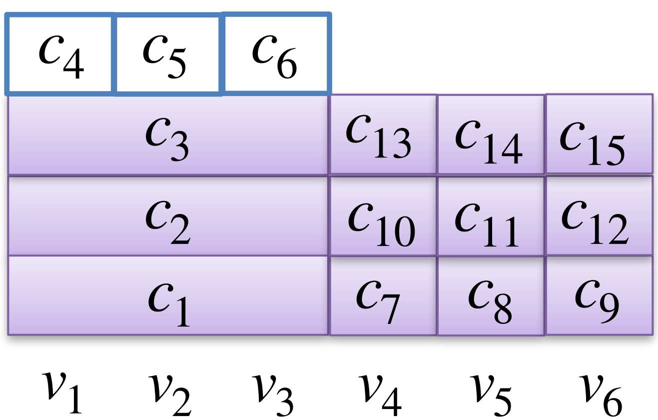

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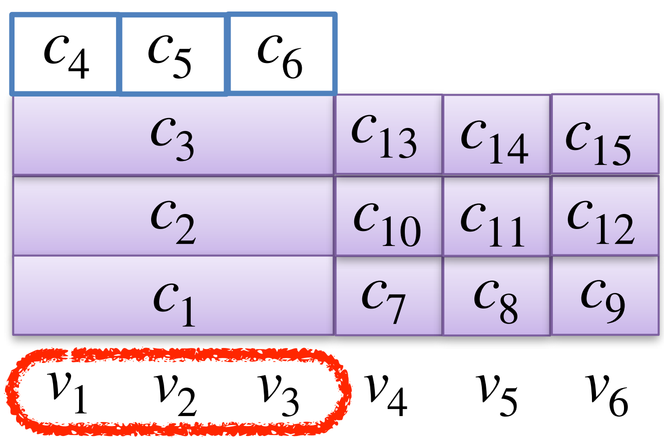

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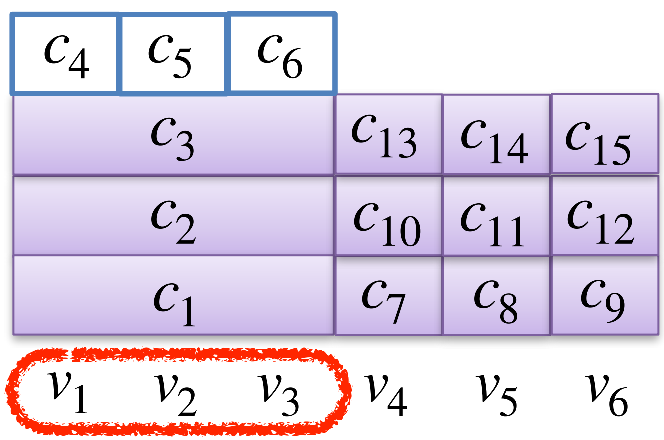

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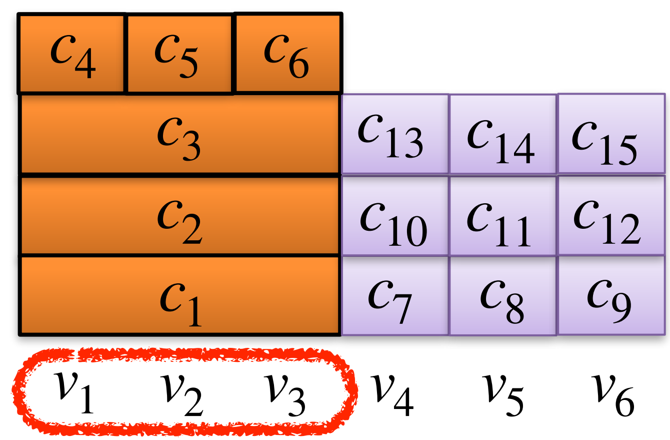

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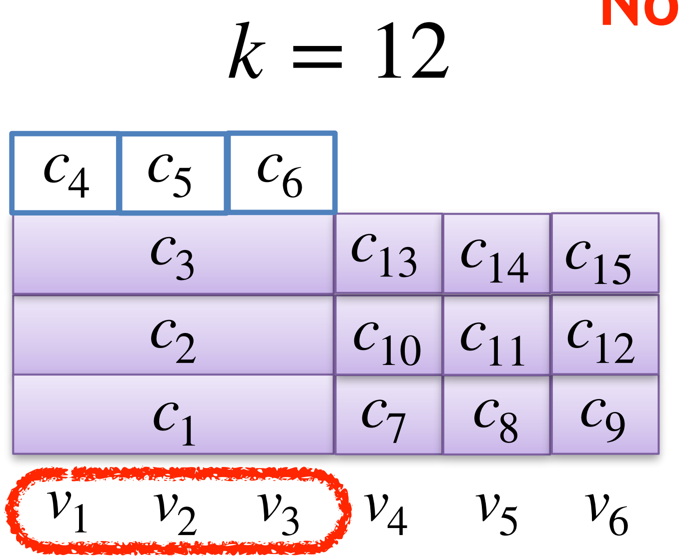
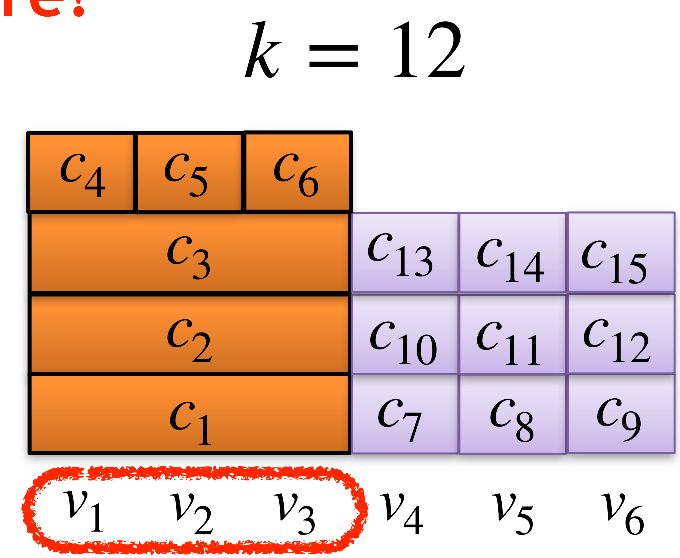
Not in the core!





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Core contradicts the Pigou-Dalton principle!

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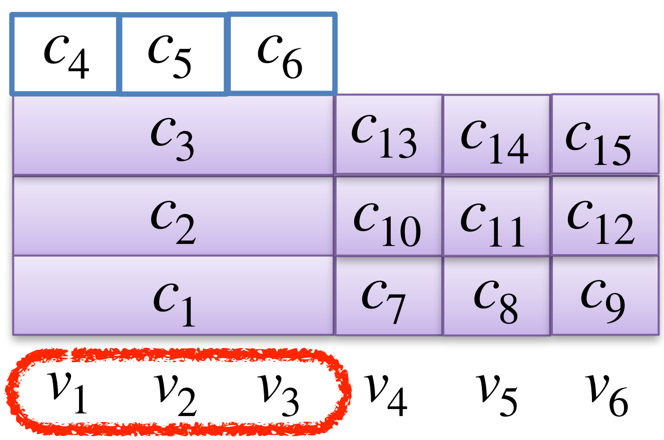

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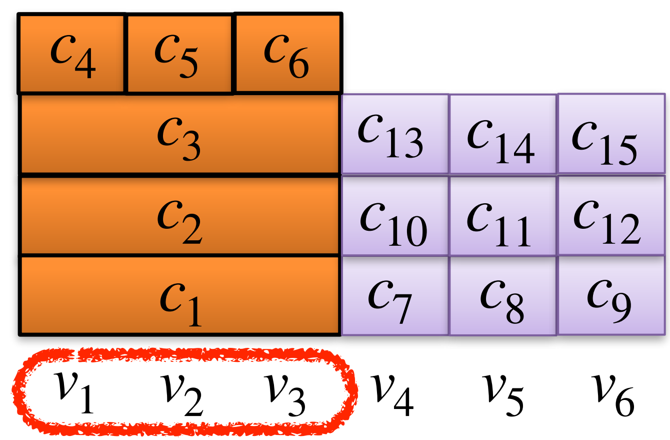

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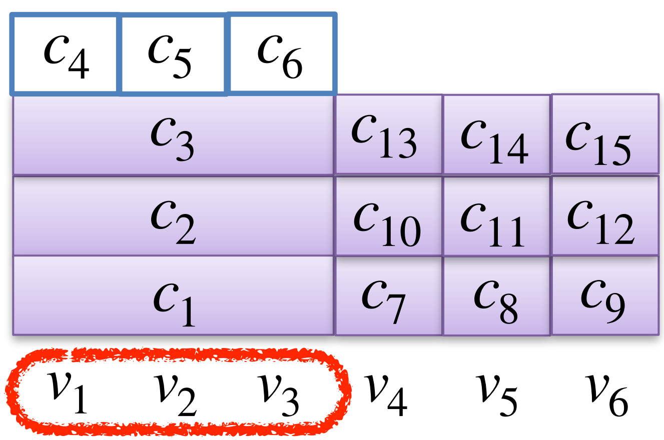


Theorem: PAV gives the best possible Approximation of the core subject to Satisfying the Pigou-Dalton principle!

Open questions:

- Does there always exist a committee in the core?
- Does there always exist a Pareto-optimal priceable committee?
- What is the best possible core-approximation among welfarist rules?

Beyond proportionality: diversity (extreme form of degressive proportionality)

Proportional Approval Voting (Thiele)

Assume voter v approves t members of a committee W. Then v gives to W the following number of points:

$$\sum_{i=1}^{t} \frac{1}{i} = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{t}$$

E.g., consider a committee 

Points per voter:

$v_1: 1+\frac{1}{2}$

$v_2: 1+\frac{1}{2}$

$v_3: 1+\frac{1}{2}+\frac{1}{3}$

$v_4: 1+\frac{1}{2}$

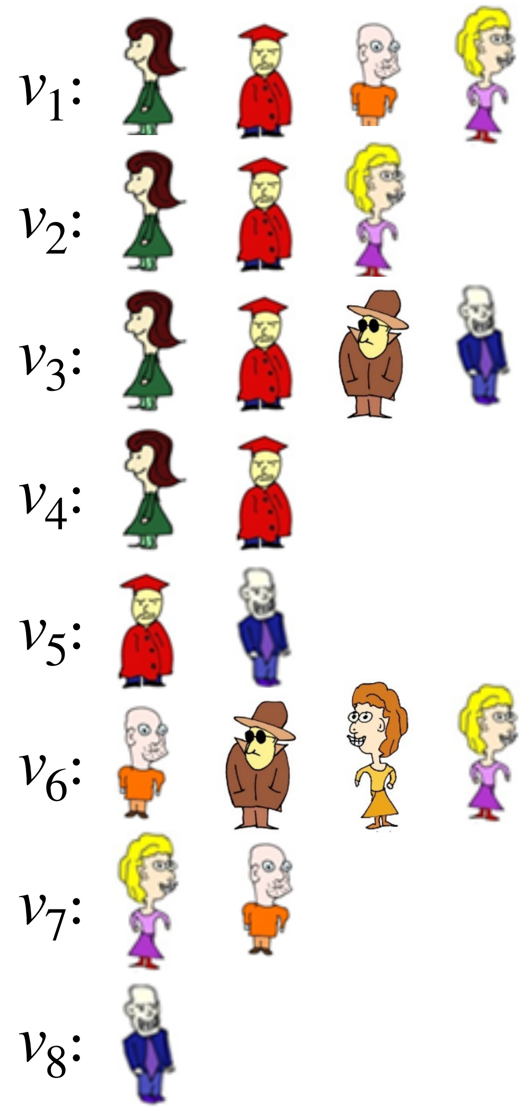
$v_5: 1+\frac{1}{2}$

$v_6: 0$

$v_7: 0$

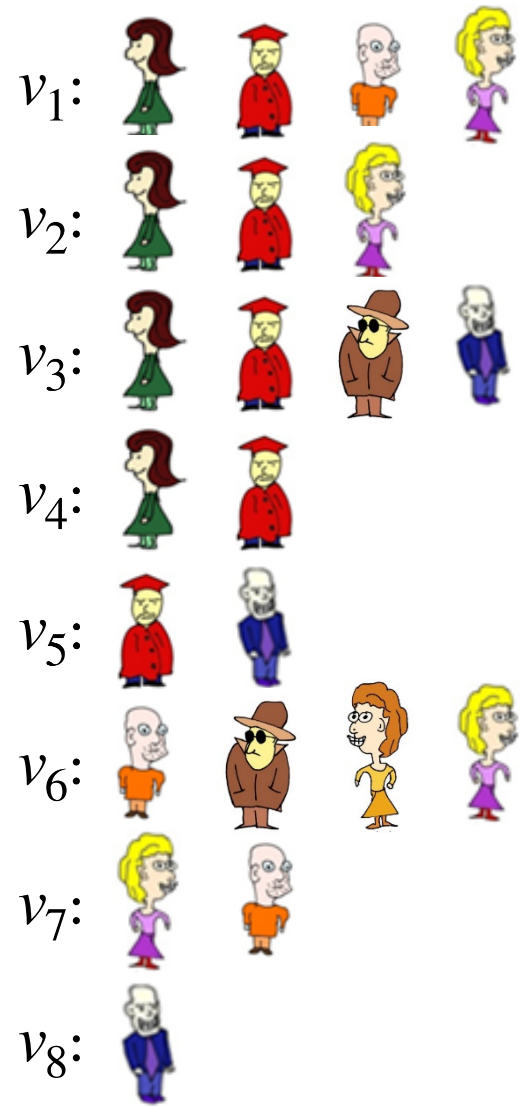
$v_8: 1$

Sum of points = $8+\frac{5}{6}$



Approval Chamberlin-Courant rule

Voter v gives to W **one** point if v approves someone from W and **zero** points otherwise.



Approval Chamberlin-Courant rule

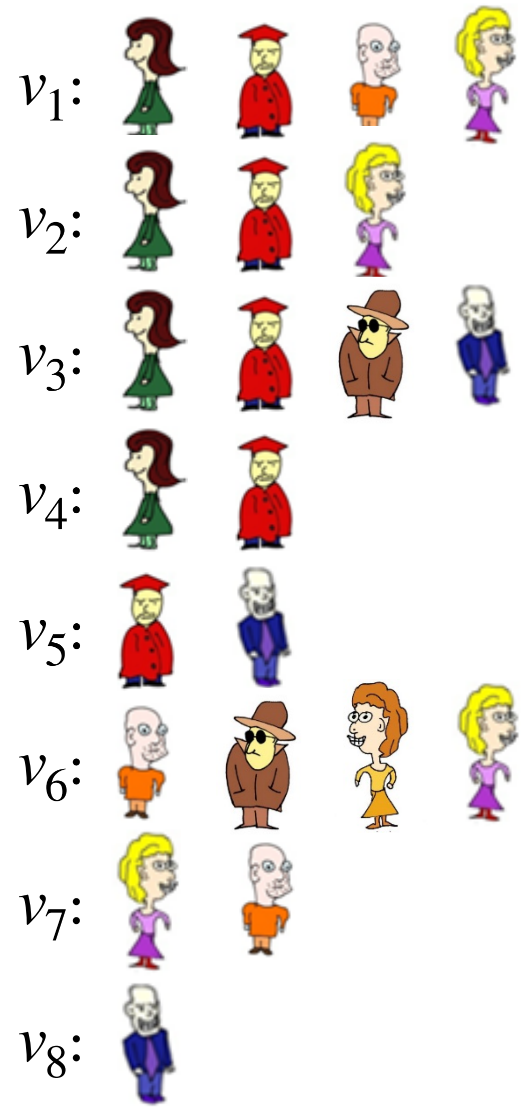
Voter v gives to W **one** point if v approves someone from W and **zero** points otherwise.

E.g., consider a committee 

Points per voter:

v_1 : 1 v_2 : 1
v_3 : 1 v_4 : 1
v_5 : 1 v_6 : 1
v_7 : 1 v_8 : 1

Sum of points = 8

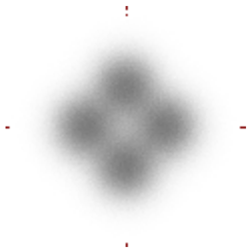


PAV for Euclidean Preferences

uniform on a circle

Gaussian

4 Gaussians



uniform on a square

[EFL+17] E. Elkind, P. Faliszewski, JR. Laslier, P. Skowron, A. Slinko, and N. Talmon: What Do Multiwinner Voting Rules Do? An Experiment Over the Two-Dimensional Euclidean Domain, AAAI 2017

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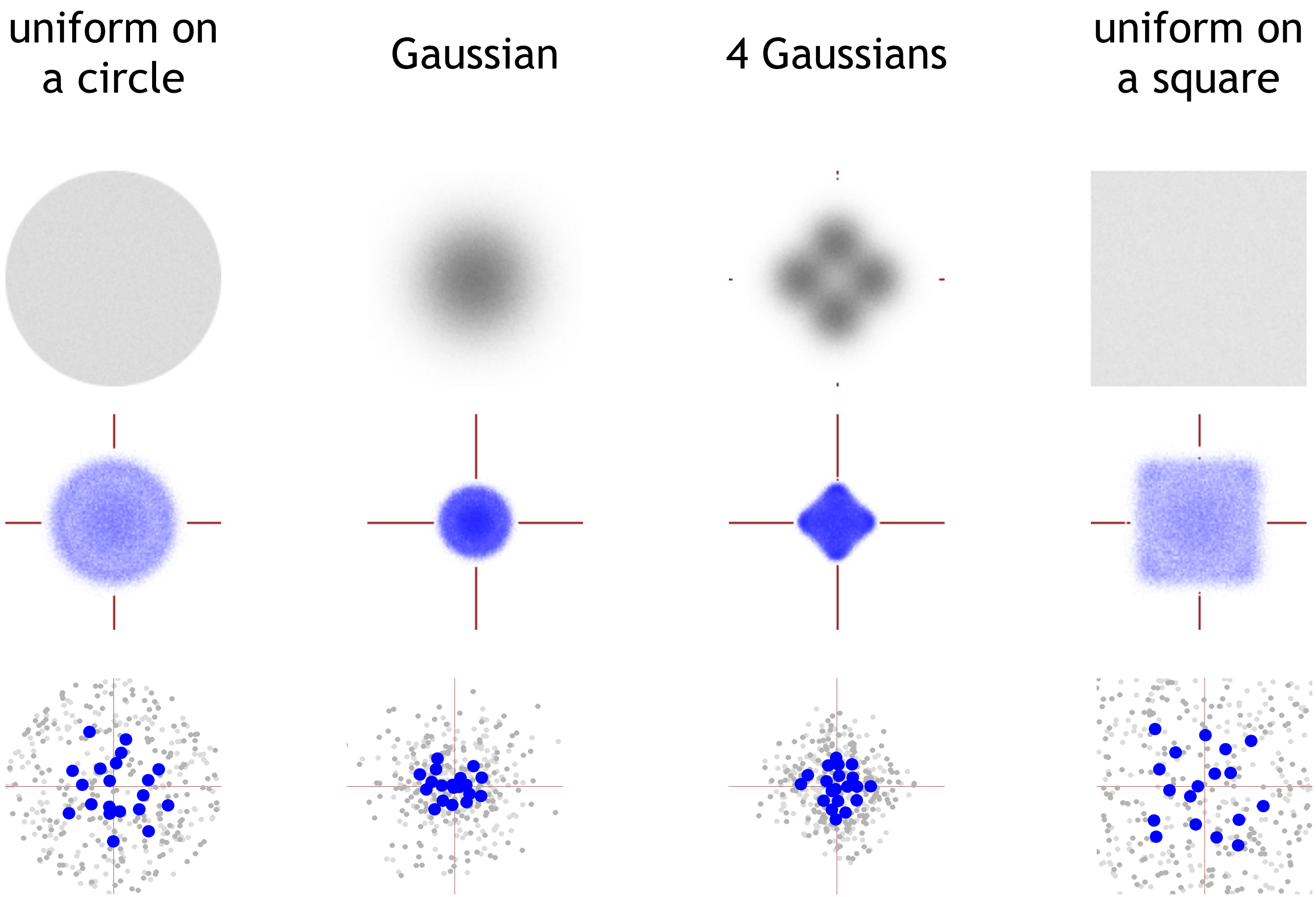

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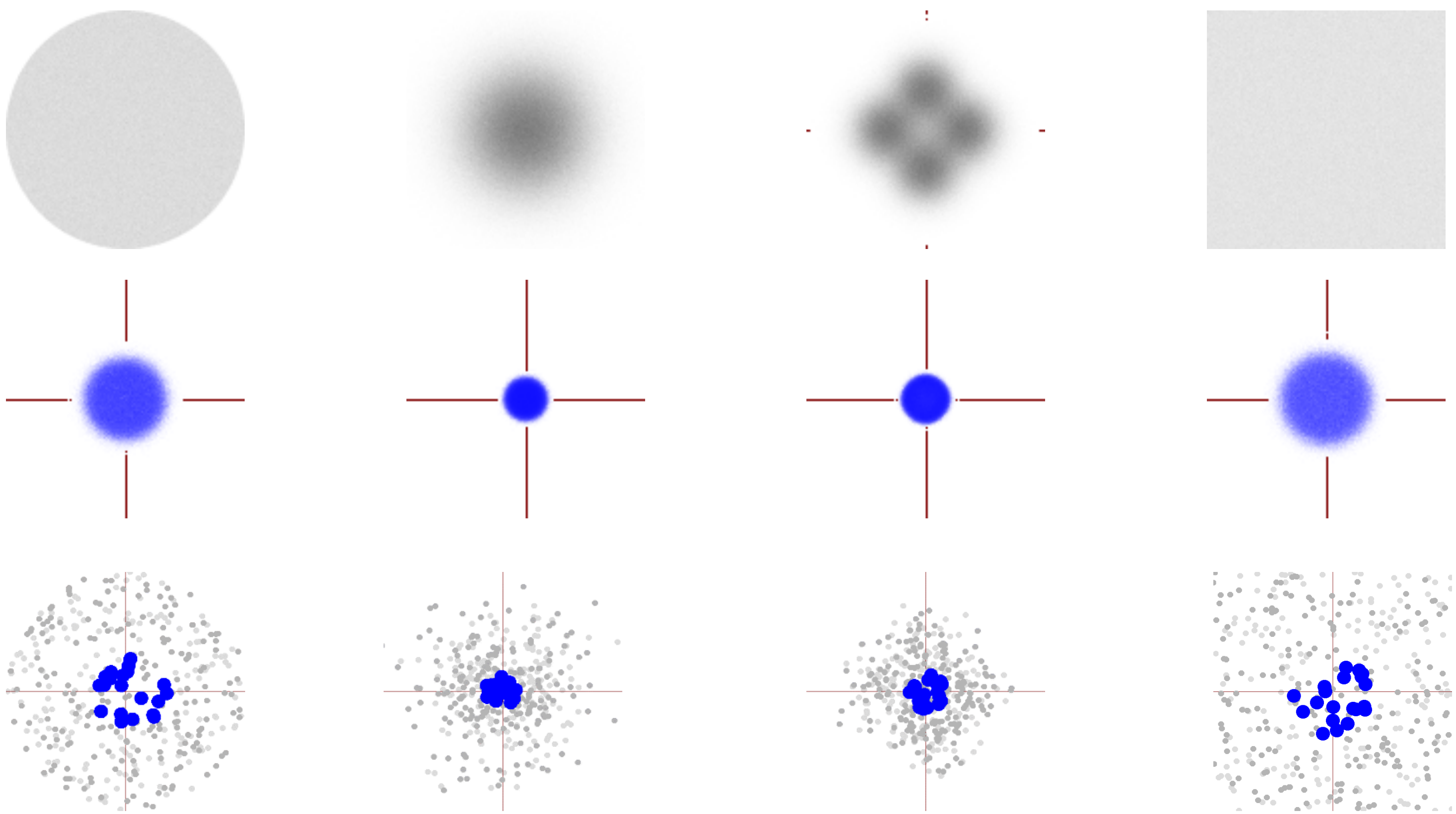
Multiwinner AV for Euclidean Preferences

uniform on a circle	Gaussian	4 Gaussians	uniform on a square



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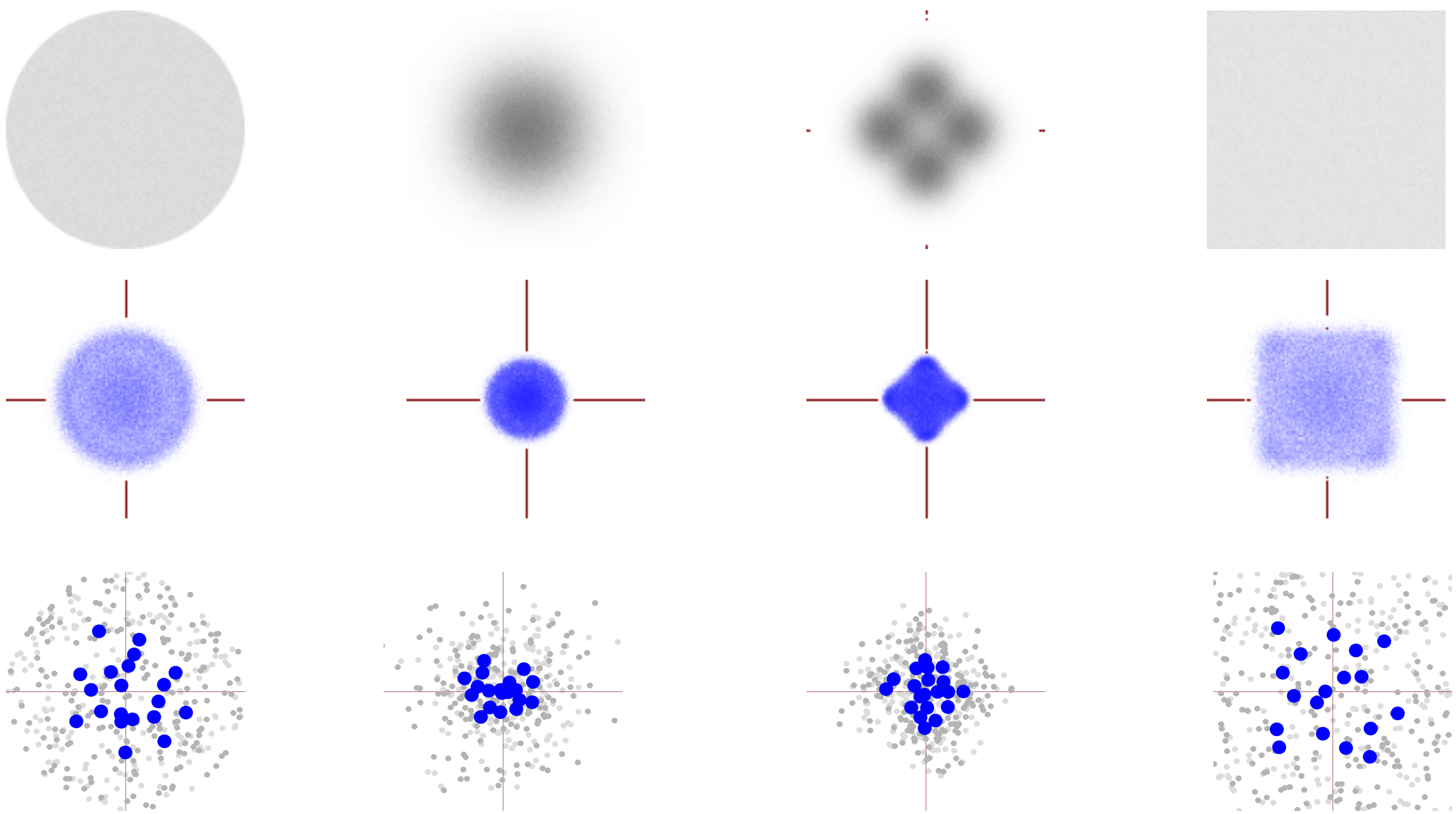

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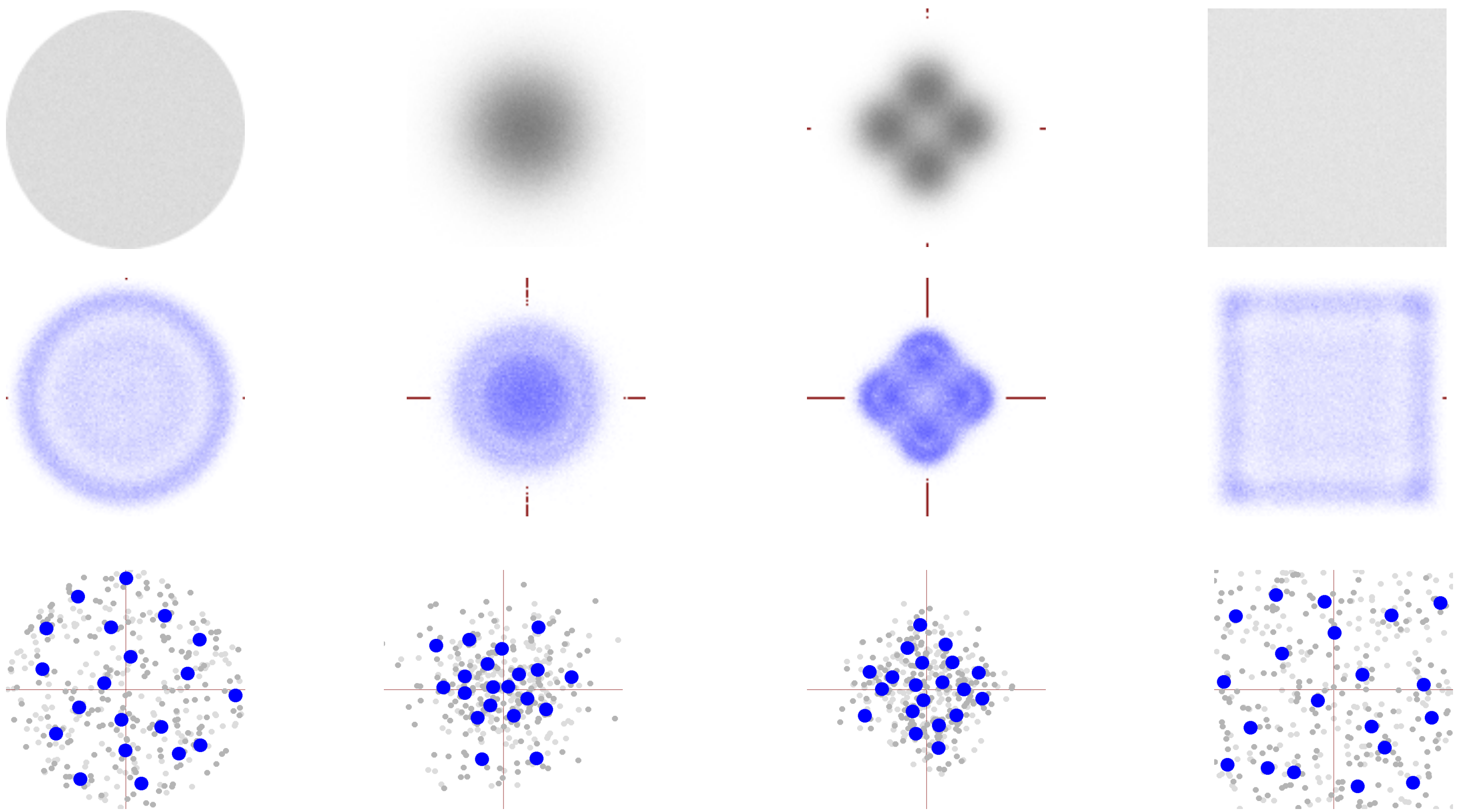

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