Final tasks
Vector Addition Systems

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1. VASS with quadratic reachability set (10pt)
Provide an example of a VASS and infinite sequence of its configurations $c_1, c_2, \ldots$ of growing maximum norm (denoted by NORM) such that the set of configurations reachable from c_i is quadratic with respect to $\text{NORM}(c_i)$.

2. Order on multisets (10pt)
Show that if $(X, \preceq)$ is a wqo then also $(\mathcal{M}(X), \preceq_M)$ is a wqo, where $\mathcal{M}(X)$ is the set of finite multisets of elements of X and an order $\preceq_M$ is defined as follows: for two multisets S_1, S_2 we have $S_1 \preceq_M S_2$ if there is an injective function $f : S_1 \to S_2$ such that for each $x \in S_1$ it holds $x \preceq f(x)$.
Hint: use the generalized Higman's Lemma, which says that if $(X, \preceq_X)$ is wqo then $(X^*, \preceq)$ is a wqo, where in the subsequence order $\preceq$ on words from X^* a letter $a \in X$ can be mapped to any letter $b \in X$ such that $a \preceq_X b$.

3. Coverability in $\mathbb{Z}$-VASS (10pt)
Show that the following coverability problem for $\mathbb{Z}$-VASSes is decidable:
GIVEN: a $\mathbb{Z}$-VASS and two its configurations $s, t \in Q \times \mathbb{Z}^d$
QUESTION: is t coverable from s, namely does there exists a path from s to some t' such that t' is at least as big as t on each coordinate?

4. 3-VASS with an exponential run (10pt)
Provide an example of three-dimensional VASS with numbers on transitions from the set $\{-1, 0, 1\}$ such that for some its states p and q the shortest run from $p(0, 0, 0)$ to $q(0, 0, 0)$ is of length exponential with respect to VASS representation size.
Hint: modify slightly the 4-VASS construction.

5. Covering diagonal in 2-VASS (10pt)
Show that the following problem for two-dimensional VASSes is decidable:
GIVEN: a 2-VASS and two its states $p, q \in Q$
QUESTION: is it true that for each $n \in \mathbb{N}$ there is a configuration $q(n_1, n_2)$ with $n_1, n_2 \geqslant n$ reachable from $p(0, 0)$?