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Exercise 1.1: Combinatory Logic

Given the rules for relativized Combinatory Logic:

$$\frac{}{\Gamma, x : \tau \vdash_{CL} x : \mathcal{S}(\tau)} \text{ (var)}$$

$$\frac{\Gamma \vdash_{CL} M : \sigma \to \tau \qquad \Gamma \vdash_{CL} N : \sigma}{\Gamma \vdash_{CL} MN : \mathcal{S}(\tau)} \ (\to \mathsf{E})$$

and

$$\Gamma = \{S : (\alpha \to \beta \to \gamma) \to (\alpha \to \beta) \to \alpha \to \gamma, \\ K : \alpha \to \beta \to \alpha\}$$

with reduction rules $SXYZ \rhd_W XZ(YZ)$, $KXY \rhd_W X$.

1. Find an inhabitant I, s.t. $IX \rhd_W^* X$ and $\Gamma \vdash I : \alpha \to \alpha$. Construct its type derivation.
2. Find another inhabitant of type $\alpha \to \alpha$ and describe its reduction and type derivation like in exercise 1.1.

Exercise 1.2: Two-Counter Automaton

Theorem 1
Let $\mathcal{A}$ be a two-counter automaton with initial configuration (q_0, n_0, m_0). $\mathcal{A}$ accepts if and only if there exists a term e with $\Gamma_{\mathcal{A}} \vdash e : q_0 \to s^{n_0}(0) \to s^{m_0}(0)$.

Lemma 1
Let $\mathcal{C}$ and $\mathcal{C}'$ be configurations in $\mathcal{A}$. We have $\mathcal{C} \to \mathcal{C}'$ if and only if there is a term e with $\Gamma_{\mathcal{A}} \vdash e : [\mathcal{C}'] \to [\mathcal{C}]$.

Lemma 2
Let $\mathcal{C}$ be a configuration in $\mathcal{A}$. If there are terms e and e' with $\Gamma_{\mathcal{A}} \vdash e : \sigma \to [C]$ and $\Gamma_{\mathcal{A}} \vdash e' : \sigma$ then $\sigma = [\mathcal{C}']$ for some configuration $\mathcal{C}'$.

Prove Theorem 1 by means of Lemma 1 and Lemma 2

Homework

Prepare your PC for exercise 4:

- Install Java 8, IntelliJ and the Scala plugin following the instructions at:
  https://www.scala-lang.org/documentation/getting-started-intellij-track/getting-started-with-scala-in-intellij.html
  Make sure you use Java 8! (Java 9 won't work!)
- Install Git `https://git-scm.com/book/en/v2/Getting-Started-Installing-Git`