PhD Open lectures, University of Warsaw
A formal framework for processes inspired by the functioning of living cells

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December 2012

Please solve the three problems below and turn in your solutions as a PDF. Please consult appropriate chapters of the lecture slides that were handed out during the lectures.

Your solutions should be sent to Bartosz Klin (`klin@mimuw.edu.pl`) by **Friday, Jan. 4, 2013**.

1. **Basic Notions** - slides 21-27.

   Consider the following reaction system with three reactions and three elements in its background set:

$$\begin{aligned} S &= \{\mathtt{a}, \mathtt{b}, \mathtt{c}\} \\ A &= \{\ (\{\mathtt{a}\}, \{\mathtt{b}\}, \{\mathtt{a}, \mathtt{b}\}), \\ &\qquad (\{\mathtt{a}, \mathtt{b}\}, \{\mathtt{c}\}, \{\mathtt{b}, \mathtt{c}\}), \\ &\qquad (\{\mathtt{c}\}, \{\mathtt{a}\}, \{\mathtt{c}\})\ \} \end{aligned}$$

   Compute the interactive processes of length 5 with the following context sequences:

$$\begin{array}{llllll} 1a) & \{\mathtt{a}, \mathtt{b}\} & \emptyset & \emptyset & \emptyset & \emptyset \\ 1b) & \{\mathtt{a}\} & \emptyset & \{\mathtt{b}\} & \{\mathtt{a}\} & \{\mathtt{a}\} \\ 1c) & \{\mathtt{a}, \mathtt{c}\} & \{\mathtt{c}\} & \{\mathtt{b}, \mathtt{c}\} & \{\mathtt{a}\} & \{\mathtt{b}\} \end{array}$$

2. **Examples** - slides 18-29.

   Consider a reaction system simulating an n-bit counter. Design a reaction system with the background

$$S = \{\mathtt{p}_1, \ldots, \mathtt{p}_n, \mathtt{collatz}, \ldots\}$$

   (you may extend the background with further elements if you wish) so that a system that represents a number k in binary, when put in the context $\{\mathtt{collatz}\}$, simulates the Collatz process on the number k, i.e., repeatedly:

- divides k by 2 if it is even,
- multiplies k by 3 and adds 1, otherwise.

A single step of the Collatz process may be simulated by several steps of your reaction system. All calculations should be done modulo 2^n, i.e., on n-bit numbers.

Hint: design reaction systems for multiplication / division by 2, and for adding two numbers represented in binary.

3. **Functions** - slides 18-30.

   For each of the following powerset functions f_1, f_2, and f_3 over $S = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, determine if the function is

   - reactant minimal?
   - inhibitor minimal?
   - resource minimal?

   and argue for your answers (using the theorems from the slides).

	f_1	f_2	f_3
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\{\mathtt{a}\}$	$\emptyset$	$\{\mathtt{b}\}$	$\{\mathtt{a}\}$
$\{\mathtt{b}\}$	$\{\mathtt{a}, \mathtt{b}\}$	$\{\mathtt{c}\}$	$\{\mathtt{a}\}$
$\{\mathtt{c}\}$	$\{\mathtt{a}, \mathtt{c}\}$	$\{\mathtt{a}\}$	$\{\mathtt{a}, \mathtt{b}\}$
$\{\mathtt{a}, \mathtt{b}\}$	$\{\mathtt{a}\}$	$\{\mathtt{b}\}$	$\{\mathtt{a}, \mathtt{b}\}$
$\{\mathtt{a}, \mathtt{c}\}$	$\{\mathtt{a}, \mathtt{c}\}$	$\{\mathtt{a}, \mathtt{b}\}$	$\{\mathtt{a}\}$
$\{\mathtt{b}, \mathtt{c}\}$	$\emptyset$	$\{\mathtt{c}\}$	$\{\mathtt{a}, \mathtt{b}\}$
$\{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$	$\emptyset$	$\emptyset$	$\emptyset$