Problem 1: In the reset streaming model, the goal of the algorithm is to maintain some statistic over an m-dimensional vector x (initialized to 0) under an arbitrary n-length sequence of the following operations:

- Increments: $x_i = x_i + 1$
- Resets: $x_i = 0$

Show that any (possibly randomized) algorithm that 2-approximates $\|x\|_0$ in this model must use $\Omega(n)$ space, even if $m = \Theta(n)$.

[In another words, show that there exist $C, C', C'' > 1$ such that for any $m > C''$ and $m/C < n < Cm$, any algorithm for 2-approximating $\|x\|_0$ must use at least n/C' bits of space.]

Problem 2: Provide a randomized streaming algorithm for maintaining a vector $x \in \{-n \dots n\}^m$ under a stream of n increments and decrements, such that at any time, given an index $i \in \{1 \dots n\}$, the algorithm outputs an estimate x_i^* such that

$$|x_i^* - x_i|^2 \leq 0.01 \sum_{i \neq j} x_j^2$$

with probability at least 2/3. The algorithm should use only $\log^{O(1)}(n+m)$ bits of space (ignoring the space needed to store random numbers).

Problem 3: Consider an orthonormal basis matrix B in R^n (the basis vectors are the columns of B). We say that B is *incoherent* if for each of its columns b, we have $\|b\|_\infty \leq O(1)/\sqrt{n}$. Also, we say that a vector $x \in R^n$ is (k, M)-*sparse* in B if $x = Bv$, where v is k-sparse, and all coefficients of v are integers in the range $\{-M, ..., M\}$.

Show that for any incoherent basis B there exists a set of coordinates $S \subset \{1...n\}$ of size $(k + \log n + \log M)^{O(1)}$, with the following property: for any vector x that is (k, M)-sparse in B, given the projection $x_{|S}$ of x on S, it is possible to recover x. The running time of the recovery algorithm is not important, as long as it is finite.