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Problem 1:

- Let $\mathcal{G}$ be a class of graphs closed under operation of taking minors, i.e. for every $G \in \mathcal{G}$ all minors of G are also in $\mathcal{G}$. Show that the following are equivalent.

    1. $\mathcal{G}$ is a class of graphs of bounded treewidth, i.e. there is a constant c such that the treewidth of all graphs from $\mathcal{G}$ is at most c;
    2. There is a planar graph H that does not belong to $\mathcal{G}$.

Problem 2:

A graph class $\mathcal{G}$ has *bounded local treewidth*, if there exists a function $f : \mathbf{N} \to \mathbf{N}$ such that for every graph $G \in \mathcal{G}$, every integer $r \in \mathbf{N}$, and every vertex $v \in V(G)$, the subgraph of G induced by vertices of distance at most r from v is of treewidth at most $f(r)$. For example, graphs of vertex degree at most 3 are of bounded local treewidth because every subgraph of radius r of such a graph has at most $3^r + 1$ vertices, and thus of treewidth at most $f(r) = 3^r$. On the other hand, a class of graphs containing arbitrarily large cliques is not of bounded local treewidth.

- Show that planar graphs are of bounded local treewidth.
- An apex graph is a graph which can be turned into a planar graph by removing at most one vertex. Let $\mathcal{G}$ be a class of graphs closed under operation of taking minors. Show that the following are equivalent.

    1. $\mathcal{G}$ is of bounded local treewidth;
    2. There is an apex graph H that does not belong to $\mathcal{G}$.

Hint: Use the grid theorem for planar and H-minor-free graphs.

Problem 3: In the parameterized CONNECTED DOMINATING SET problem (CDS) we are given a graph G and a parameter k. The question is if G contains a dominating set of size at most k inducing a connected subgraph of G.

1. Show that on planar graphs with n vertices the problem is solvable in time $2^{O(\sqrt{k}\log k)} n^{O(1)}$;
2. Improve the running time of your algorithm to $2^{O(\sqrt{k})} n^{O(1)}$.

Problem 4: Describe an EPTAS for CDS on planar graphs.