PhDOpen, Warsaw

Evaluating Formulas in Sparse Graphs

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Assignment — Due: 30 November 2010

1. We proved in the lectures that 3-colourability is NP-complete, even when restricted to graphs of degree at most 5. In this exercise, you are asked to do the same, for graphs of degree at most 4. That is, prove that on $\mathcal{D}_4$, 3-colourability is NP-complete.

   Also show that every graph in $\mathcal{D}_3$ is 4-colourable.

2. It follows from the exercise above that if 3-colourability was definable in FO on graphs of degree 4, then P = NP. Explain why.

   But, we can show more. Prove that 3-colourability is *not* definable in FO on graphs of degree 4. For instance, you can use Gaifman's locality theorem to prove this.

3. The *crossing number* of a graph G is the least number k such that there is a an embedding of G in the plane with at most k points where edges cross. In particular, planar graphs have crossing number 0.

   Show that the class of graphs with crossing number k has bounded local treewidth.

   Conclude from this that for any class that has *locally bounded crossing number* FO satisfaction is FPT.

4. Show that the independent set problem is FPT on classes of graphs that are nowhere-dense. That is, for any nowhere-dense class $\mathcal{C}$, there is an algorithm A that determines whether a graph $G \in \mathcal{C}$ contains an independent set of k elements and the running time of A is bounded by $f(k)|G|^c$ for some constant c.

   What can you say about the *clique* problem?

5. Let $\mathcal{E}$ be the class of graphs which are disjoint unions of cliques. That is, the vertices of each graph $G \in \mathcal{E}$ can be partitioned into cliques in such a way that there are no edges between the parts.

   Show that satisfaction of FO is FPT on $\mathcal{E}$.