Homework for PhD Open

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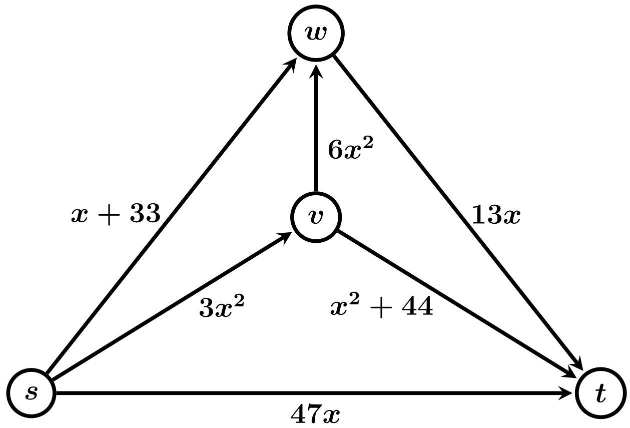
1 TFNP

Show that if a problem in TFNP is FNP-hard then FNP = co-FNP.

2 Equilibira of congestion games and bimatrix-games

1. Construct two 2x2 bimatrix games. Both should have **exactly two equilibria** (pure or mixed). For one game, ensure that only pure strategy profiles on the diagonal can be realised in equilibrium; for the other game, ensure that only pure strategy profiles off the diagonal can be realised. Hint: consider the first example in the slides on PLS.

2. Consider the following two-player routing game, with latency functions shown on the edges. Both players want to go from s to t. They have weights w_1, w_2 respectively (which is how much they contribute to the latency function if they use an edge).



Consider two cases: (i) $w_1 = 1, w_2 = 2$ (weighted); (ii) $w_1 = w_2 = 1$ (unweighted). For each case, convert the game to a bimatrix game and compute all equilibria (pure and mixed). Show your working. Hint: For case (i), you can dramatically simplify the game with *iterated elimination of strictly dominated strategies*.

3 Polymatrix and congestion games

In a *team* polymatrix game, every bimatrix game on every edge has identical payoff matrices for both players.

1. Provide a potential function for team polymatrix games, thereby showing that they always have at least one pure equilibrium.

2. Reduce local max cut to the problem of finding a pure Nash equilibrium in a team polymatrix game.

3. Reduce the problem of computing an equilibrium (mixed or pure) of a team polymatrix game to that of computing an equilibrium in a (general) congestion game.

4. Recall that computing the mixed Nash equilibrium of a general congestion game is CLS-complete. Why is the problem of computing a mixed Nash equilibrium of the party affiliation game (i.e. the game view of local max cut) not a good candidate for being CLS-hard? Hint: present an efficient algorithm to find a mixed equilibrium.

4 2-player turn-based zero-sum discounted games

For the 2-player zero-sum discounted games presented in the lectures, reduce the problem of solving them to that of finding the fixed point of a contraction map under the l_∞ norm.