Open lectures for PhD students in computer science
Combinatorial limits course by D. Král' and A. Grzesik
Assignment #2

1. Express  in terms of 4-vertex rooted flags.

2. Consider a convergent sequence of graphs, where vertices of each n-vertex graph are of degree $n/3$ or $2n/3$. Prove that in the limit the sum of densities of  and  is equal to $1/3$.

3. Using the inequality $[\![($  $-$  $)^2]\!]_\bullet \geq 0$ prove Mantel's Theorem for limits, i.e., if  $= 0$ then  $\leq 1/2$.

4. By considering the sequence of blow-ups of a hypothetical counterexample, prove Mantel's Theorem, i.e, each n-vertex triangle-free graph has at most $n^2/4$ edges.

5. Using the Cauchy-Schwarz inequality $[\![$  $]\!]_\bullet^2 \leq [\![$  $^2]\!]_\bullet$ prove Goodman's bound, i.e.,  $\geq$  $(2$  $- 1)$.

Open lectures for PhD students in computer science
Combinatorial limits course by D. Král' and A. Grzesik
Assignment #2

1. Express  in terms of 4-vertex rooted flags.

2. Consider a convergent sequence of graphs, where vertices of each n-vertex graph are of degree $n/3$ or $2n/3$. Prove that in the limit the sum of densities of  and  is equal to $1/3$.

3. Using the inequality $[\![($  $-$  $)^2]\!]_\bullet \geq 0$ prove Mantel's Theorem for limits, i.e., if  $= 0$ then  $\leq 1/2$.

4. By considering the sequence of blow-ups of a hypothetical counterexample, prove Mantel's Theorem, i.e, each n-vertex triangle-free graph has at most $n^2/4$ edges.

5. Using the Cauchy-Schwarz inequality $[\![$  $]\!]_\bullet^2 \leq [\![$  $^2]\!]_\bullet$ prove Goodman's bound, i.e.,  $\geq$  $(2$  $- 1)$.