

# Exercises

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## 1 Exercise 1

Fill in the following table by YES or NO.

	Intersection	Complement	Morphisms	Product	Quotients
Rational languages are closed under					
Star-free languages are closed under					
Commutative languages are closed under					

## 2 Exercise 2

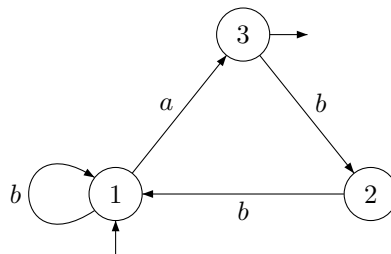
Let  $A = \{a, b\}$ . Indicate, for each of the languages  $L_1, L_2, L_3, L_4$  whether it is rational, star-free or commutative.

- (1)  $L_1 = bA^*abA^* \cap A^*bbA^*$ ,
- (2)  $L_2 = \{u \in A^* \mid |u| \equiv 2 \pmod{5}\}$
- (3)  $L_3 = (A^2)^*(a + bb)$ ,
- (4)  $L_4 = A^*(ab + ba)A^*$ .

Briefly justify your answers.

## 3 Exercise 3

Let  $L$  be the language accepted by the following partial automaton



Compute the syntactic monoid of  $L$ . If you fill comfortable, you can also compute its syntactic order.

Is this monoid aperiodic?

Show that  $L$  is star-free and give a star-free expression for representing  $L$ .

Give a first-order formula defining  $L$ .