PROBLEMS

1. Consider a merge junction $\mathcal{A}$ with two incoming roads "1" and "2" (with parameters v_1, k_1 and v_2, k_2 respectively) and one outgoing road "3" (with parameters v_3, k_3). Assume road conditions permit all parameters to range between 40 and 80. Assume that the constraints for no-backups hold (page 33 in Part 1 of slide presentation). The minimum and maximum permitted flows through $\mathcal{A}$ are therefore 3200 and 6400, respectively. Write appropriate constraints so that $\mathcal{A}$ satisfies the following additional properties:

   (a) The density k_1 is restricted to range between 40 and $\max\{40, \lceil k_2/2 \rceil\}$, inclusive.

   (b) Introduce time-varying parameters, i.e., $v_1(t), k_1(t), v_2(t), \ldots$ where t ranges over discrete time units $0, 1, 2, \ldots$. Assume it takes one time unit to move traffic accross the junction.

   The velocity $v_3(t+1)$ along the outgoing road is adjusted so that, at all times t, the flow is equal to $\max\{5000,\ v_1(t) \cdot k_1(t) + v_2(t) \cdot k_2(t)\}$. In other words, the outgoing flow along road "3" is never allowed to go below 5000.

   Hint: Try to complete the following
   if $v_1(t) \cdot k_1(t) + v_2(t) \cdot k_2(t) < 5000$
   then $v_3(t+1) = \ldots$
   else $v_3(t+1) = \ldots$

   (c) Introduce a time-varying *buffer zone* $b(t)$ for incoming road "1".

   The buffer releases the accumulated traffic from road "1" and allows it to enter the junction only when it has reached at least 50, in which case it enters the junction with velocity 40.

   (d) Consider the tiny traffic network on page 41 of Part 1. Introduce additional constraints of the form suggested in (a), (b) and (c) above at the merge junction M. Infer types for the two entry roads ("1" and "4") and the two exit roads ("4" and "5") so that these additional constraints are satisfied. Keep in mind that we want the inferred types to be time-independent. Start the inference process by choosing the type at exit "5" as "strong" as possible.

2. For a given traffic module $\mathcal{A}$ consider the set $\mathcal{T}$ of all its typings (pages 4-7 in Part 2 of slide presentation).

   Argue convincingly (a formal proof is more complicated) that $\mathcal{T}$ is a partially ordered set under the $<:$ ordering, with finitely many minimal members and finitely many maximal members.

3. Show that with rule SUBTYPING (page 21 in Part 2 of slide presentation), all three adjustments on page 20 are equivalent.

4. Prove the two parts of **Fact** on page 23 in Part 2 of slide presentation.

5. Prove the two parts of **Fact** on page 10 in Part 3 of slide presentation. For the second part, when the network is finite, determine an upper bound on the time complexity of the algorithm.

6. Show that the networks on pages 18 and 20 in Part 3 of slide presentation are inherently unstable, i.e., they have no stable configurations, and that the network on page 19 has exactly one stable configuration, namely the one shown on the page.

7. Prove that if the underlying graph of the network is acyclic, the network has stable configurations, regardless of how many routing policies are used. Devise an algorithm that constructs a stable configuration for such a network.