Exact and approximate distances in graphs
Uri Zwick

Exercises to be solved to get a grade

Solve as many of the exercises below as you can. E-mail the solutions, preferably as a pdf file, to zwick@tau.ac.il by May 31, 2007.

1. Obtain an $O(n^\omega \log n)$-time algorithm for computing the *diameter* of a directed unweighted graph on n vertices.

2. Obtain a version of Seidel's algorithm that uses only *Boolean* matrix multiplication. (Hint: Consider distances also modulo 3.)

3. Suppose that distances in a directed unweighted graph on n vertices can be computed in $T(n)$ time. Show that distances in a directed graph with integer weights of absolute value at most M can be computed in $O(T(2Mn))$ time.

4. Let $G = (V, E)$ be an n-vertex graph with integer edge weights of absolute value at most M. Describe an efficient algorithm for computing all distances in G that are at most N, for some parameter N.

5. Obtain a variant of the query answering algorithm of Yuster and Zwick that given a guarantee that a certain distance is realized using a path composed of at at least s edges can report this distance in $O(n \ln n/s)$ time.

6. The girth of a graph is the size of the shortest cycle in the graph. Show that the girth of any n-vertex graph with at least $n^{1+1/k}$ edges is at most $2k$.

7. Let $G = (V, E)$ be an undirected unweighted graph. A *weighted* graph $G = (V, E')$ is said to be a t-emulator of G if and only if for every $u, v \in V$ we have $\delta_G(u, v) \leq \delta_{G'}(u, v) \leq t\,\delta_G(u, v)$. (Note that G' is not necessarily a subgraph of G.) Show that every n-vertex graph has a 4-emulator with $O(n^{4/3})$ edges.

8. What is the maximum stretch of the variant of the query answering algorithm of Thorup and Zwick that does not swap u and v in each iteration, i.e., finds the smallest i for which $w = p_i(u) \in B(v)$ and returns $\delta(u, w) + \delta(w, v)$?