# PhD Open lectures, University of Warsaw

## The Univalence Axiom in Dependent Type Theory

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Please solve the three problems below and turn in your solutions as a PDF. Slide numbers below refer to the PDF document available from the course webpage.

Your solutions should be sent to Bartosz Klin (`klin@mimuw.edu.pl`) by **Friday, Jan. 13, 2017**.

1. Do the exercise on slide 4 with the HOL definition of $Eq_A$, both classically (using the Law of Excluded Middle at one place) and constructively (not using LEM).

2. Prove the groupoid laws 3–6 on slide 14 using $transp_B$ and $bpi_C$ from slide 13. (Note that both have implicit arguments $a, x : A$.)

3. Define *idtoEquiv* on slide 17 using $transp_B$ and $bpi_C$ from slide 13.

   There are actually several ways of doing this. In the following the underscore symbols $\_$ stand for proof terms that do not interest us. Try to find a definition of *idtoEquiv* with the following property:

   For all $e = (f, \_) : A \simeq B$, we have that

   $$ua(e) = ((p, \_), \_) : Contr(Fib_{idtoEquiv}(e))$$

   with $p : Eq_U(A, B)$ such that transport along $p$ in the type family $B : U \to U$ defined by $B(X) = X$ is equal to the function $f : A \to B$. In words, if $f$ is the equivalence from $A$ to $B$, then transport along the path given by $ua$ is just the function $f$ itself.