# Open lectures for PhD students in computer science <br> Combinatorial limits course by D. Král' and A. Grzesik <br> Final exam problems 

1. Consider $p \in[0,1]$ and a graphon $W$ such that $W(x, y)=p$ if $x, y \in\left[1-2^{-n+1}, 1-2^{-n}\right]$ for some $n \in \mathbb{N}$, and $W(x, y)=0$ otherwise. For any $k \in \mathbb{N}$ determine the density of $K_{k}$ in $W$.

2. Prove the inequality $\circ \leq 3 \Omega+3 / 8$ and argue that any extremal graphon satisfies $\mathcal{\circ}=1 / 4$ for almost all possible placements of the root.
3. Describe all graphons satisfying the equalities $\llbracket(\delta-1)^{2}(\delta-1 / 2)^{2} \rrbracket .=0$ and $\delta=3 / 4$.
