Open lectures for PhD students in computer science Combinatorial limits course by D. Král' and A. Grzesik Final exam problems

1. Consider $p \in [0,1]$ and a graphon W such that W(x,y) = p if $x, y \in [1-2^{-n+1}, 1-2^{-n}]$ for some $n \in \mathbb{N}$, and W(x,y) = 0 otherwise. For any $k \in \mathbb{N}$ determine the density of K_k in W.



2. Prove the inequality $\stackrel{\circ}{\sim} \leq 3 \& + 3/8$ and argue that any extremal graphon satisfies & = 1/4 for almost all possible placements of the root.

3. Describe all graphons satisfying the equalities $[(\mathcal{L} - 1)^2(\mathcal{L} - 1/2)^2]_{\bullet} = 0$ and $\mathcal{E} = 3/4$.