PhD Open lectures, University of Warsaw

Computation Theory with Atoms

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Please solve the problems below and turn in your solutions as a PDF. To get a grade n, you need to solve $n-1$ problems.

Your solutions should be sent to Bartek Klin (`klin@mimuw.edu.pl`) by **Wednesday, May 31, 2017**.

All problems below concern sets and structures with *equality* atoms, discussed in almost the entire course.

1. Show languages $K, L \subseteq \mathbb{A}^*$, recognizable by deterministic register automata, such that the language
$$KL = \{vw \mid v \in K, w \in L\}$$
is not recognized by any deterministic register automaton.

2. Prove that every equivariant function between equivariant single-orbit sets with atoms is surjective.

3. Prove that if X and Y are (equivariant) orbit-finite sets then $X \times Y$ is orbit-finite too. Show that, however, the number of orbits in $X \times Y$ can be arbitrary large, even if X and Y are single-orbit sets.

4. For any equivariant set with atoms X and any finite set $S \subseteq \mathbb{A}$ of atoms, let $X_S \subseteq X$ denote the set of all elements of X that are supported by S. Prove that if $|S| = |S'|$ then there is a bijection between X_S and $X_{S'}$. What can you say about the support of such a bijection?

5. Assume an equivariant set X such that every finite set S of atoms supports only finitely many elements of X. (In the lecture we proved that every orbit-finite set has this property). Using Problem 4 above, for every natural number n let $f(n)$ be the number of elements of X supported by a set of n atoms. Prove that X is orbit-finite if and only if $f(n)$ grows polynomially in n, i.e., that $f(n) = \mathcal{O}(n^k)$ for some number k independent from n.