A Toolbox for Online Algorithms
The Exam

Problem 1. Consider the static list reorganization (i.e., input consisting only of SEARCH requests). Upon a SEARCH request of the element x which is at position i on the list, the algorithm $\mathrm{MTF}_{1/2}$ moves x $\lceil (i-1)/2 \rceil$ positions towards the beginning of the list. Show that such algorithm is $O(1)$-competitive.

Problem 2. Consider the following deterministic algorithm WFA for the file migration problem on two nodes (connected by an edge of length 1). At step t, let x_t be the node holding the file of WFA and let y_t be the opposite node. Then, at the end of step t, the algorithm migrates the page to y_t iff $w_t(y_t) + D = w_t(x_t)$, where w_t is the work function. Is WFA $O(1)$-competitive? Prove or disprove.

Problem 3. Consider the file migration problem on a general graph and the following randomized algorithm FLIP. In step t, upon seeing a request at r_t, FLIP serves the request and then migrates the file to r_t with probability $\frac{1}{2D}$. In this context, D is the file size. Show that FLIP is 3-competitive.

Hint: Construct a potential function and use the triangle inequality of the distance function.

Send your solutions in pdf (in English or Polish) till the 28th of April by email (you may find my email on the webpage http://www.ii.uni.wroc.pl/~mbi/).

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